

Discrete Choice Models of Demand with Reference Dependent Utility

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This article extends the standard discrete choice framework to estimate demand in markets with differentiated products when consumers display reference dependent (R-D) utility. In industries with a common reference product, R-D preferences generate novel substitution patterns that cannot be rationalized in the standard framework, making the R-D model testable. Such substitution patterns are also present with relatively low heterogeneity among consumers' reference product. As the level of heterogeneity increases, the difference between the R-D predictions and standard predictions decreases. When the true data generating process comes from R-D preferences, simulations show that standard models deliver biased estimates of elasticities. Finally, using consumer scanner data, I reject the hypothesis that the facial tissue industry contains no R-D consumers, and estimate that 25 percent of Kleenex's market share derives from its status as the reference brand.

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One of the most robust findings of the behavioral economics literature over the past 30 years is the importance of reference dependence: the idea that people

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evaluate outcomes by comparing them to a reference point, caring more about losses than gains.¹ This concept applies quite naturally in product markets, but little work has been done to incorporate reference dependence into models of consumer demand. One reason for this may be that conventional models are believed to be flexible enough to accommodate behavioral biases like reference dependence, even without explicitly modeling these biases.² This paper shows this is not true. Standard models, such as logit models with random coefficients, cannot rationalize the substitution patterns generated by consumers with reference-dependent preferences, and ignoring consumers' biases can lead the empirical economist to wrong conclusions.

My main goal is to extend the conventional discrete choice framework to account for reference dependent (R-D) preferences in markets with product differentiation. To pursue this goal, I incorporate the R-D preferences theory of Koszegi and Rabin (2006) into a broad class of discrete choice models originated by the work of Mcfadden (1981) and later extended by Berry et al (1995, 2004) (henceforth BLP). In this class of models, a consumer's indirect utility depends only on product characteristics and individual tastes. In the framework I propose, I also allow consumers to obtain asymmetric payoffs as functions of gains and losses with respect to a reference point.

Experimental evidence suggests that consumers may form reference points in accordance to expectations, aspiration levels, publicity, norms, and social comparisons (Baucells et al 2011). Incorporating reference points into the analysis of demand helps explain consumer unwillingness to change products after experiencing a quality decrease (status quo bias). Since R-D preferences induce loss aversion, the product's own price elasticities might be larger for a price increase than for a price decrease, a discrepancy that affects how firms choose their pricing strategies. Given that reference points can be generated by publicity, firms have incentives to advertise

¹See the surveys of : Camerer et al (2011), DellaVigna (2009), Kahneman (2003) or Mullainathan and Thaler (2000).

²See the discussions presented in: Camerer and Malmendier (2017), Ellison(2006) and Hendricks(2006).

in order to place their product as the reference in the industry.

Previous research in industrial organization has shown that models that account for consumer heterogeneity (such as BLP) are flexible enough to accommodate rich substitution patterns.³ This class of models is a key element in merger simulations and in many policy counterfactuals. In two different scenarios, I show that standard models fail to predict important substitution patterns, and thus could generate opposite predictions than those generated under the R-D model. The first scenario (Proposition 1) assumes that consumers share a common reference product in the industry; the second scenario (Proposition 2) relaxes this assumption and studies the possibility of heterogeneity in the reference points among consumers.

Assuming a common reference point implies that consumers agree on which product serves as the standard for comparing other products. It is not a mere simplification but rather a possible description for several industries. For example the marketing literature identifies the following industries: Smartphones (iPhone), laundry detergent (Tide), cream cheese (Philadelphia), eReaders (Kindle) or live action cameras (GoPro).⁴ From the researcher's point of view it could be challenging to specify which product is the reference product. As a first step, I provide a simple way for testing that the data generating process contains a common reference product.

For most industries, a common reference product among consumers does not exist. It is possible to model R-D preferences in this context if we are willing to assume a mechanism that reveals which is the reference products for each consumer. In this case, I show that the model can still produce novel substitution patterns when the number of reference products is sufficiently small. However as the number of reference products increases, the R-D model generates similar substitution patterns as those predicted by standard models.

Having established that the R-D model is non-parametrically testable, and thus can explain non-standard substitution patterns, I restrict the attention to the linear

³For examples, see Nevo (2001) and Petrin (2002).

⁴See Amaldoss and He (2017) and Zhou (2011).

random coefficient model to provide a practical and estimable version of an R-D model. I then present several simulations, comparing the performance of the R-D model against standard models. I show that in a simulated world with no reference dependence, the R-D model performs as well as BLP. Nevertheless the R-D model outperforms BLP whenever reference products exist in the simulated world. As shown with the non-parametric model, when the reference product becomes similar to a competing products in my simulation, BLP predicts (wrongly) a decrease in its market share, R-D correctly predicts an increase. Since the linear BLP model combines in one coefficient the price effect and effect from loss aversion, it overestimate consumers' sensitivity to prices.

Finally, using consumer scanner data I analyze the facial tissue industry. Kleenex is a suitable candidate for a reference product. I estimate and test the R-D model by taking advantage of a long panel, and the available microdata provided in the IRI academic dataset. My estimates are consistent with reference dependence, and the idea that consumers exhibit loss aversion. I estimate that loss aversion provides the classical Kleenex 25 percent of its market share.

This research makes contributions in several fields. First it contributes to the industrial organization literature by providing a suitable model to estimate demand for industries that contain reference dependent consumers. It contributes to behavioral economics by testing its most popular theory in a real-world setting. By accounting for the endogeneity problem of prices, this research also appeals to the marketing readers where the idea of reference price has been studied in great detail. The rest of the paper is organized as follows: section 1 places this research in the context of previous literature; the R-D model and its identification results are presented in section 2; section 3 provides simulations to compare the performance of the model; section 4 describes the data for the empirical application; section 5 presents the results of it; and section 6 concludes with a discussion.

I. Relationship with previous work

In this paper R-D preferences are understood as in the model of Közsegi and Rabin (2006) (henceforth the KR model). This theoretical model generalizes previous behavioral models of R-D utility into a parsimonious theory. Loosely speaking, R-D preferences can be thought as a phenomenon when consumers care not only about the characteristics of their purchase, but also how those characteristics differ from a particular consumption bundle, called the reference. This section explains the key lessons from the KR model, and how this model emerged from early work. It also positions my research in the context of the current literature, providing a behavioral economics background for those applied economists less familiar with this literature.

Kahneman and Tversky (1979) introduced R-D preferences to explain regularities in the experimental evidence of choice under uncertainty. With respect to a reference point, agents perceive differently gains (if the payoff is greater than the reference point) and losses (if the payoff is lower than the reference point). In their model (Prospect theory) the pain of losing x dollars is greater than the pleasure of winning x dollars, i.e., agents are loss averse. Over the following years the research on loss aversion, and particularly in the R-D expected utility, became extremely popular, yet it took 21 years to be extended into the theory of riskless choice. To that end, Tversky and Kahneman (1991) model the reference point as a combination of multiple characteristics. That is, any attribute of a product is evaluated separately as a gain or as a loss with respect to the reference product.⁵ Usually the reference product was thought of as the initial or previous endowments consumers had. The theory explains common behavioral biases outside the uncertain world, such as the endowment effect and the status quo bias.

In reality, however, there are several scenarios where there is no endowment to act as the reference point. A researcher that expects to get published suffers more from

⁵Suppose that you booked a hotel room with a twin bed and no air conditioner. The manager offers you two alternative rooms, one with a queen bed and no air conditioner, and one with a full bed with air conditioner. Any offered alternative seem better than you previous room. Suppose that you are indifferent between them. Another day you are assigned to a room with a twin bed but with air conditioner. The manager offers you again the alternatives to switch rooms as before. Loss aversion implies that you will choose the full bed with air conditioner.

rejection than one that expects to be rejected. In a more general sense reference points can be generated by expectations, aspiration levels, publicity, norms, and social comparisons. To account for that, the the KR model extends the notion of a reference point to beliefs that consumers have over possible outcomes. For example, a consumer might believe that with probability $1/2$ the price of a beer will be 10 dollars, and with probability $1/2$ the price will be 5 dollars. Then a beer that is 7 dollars feels like a gain with respect to the possibility of paying 10 dollars, but as a loss to the possibility of paying 5 dollars. The KR model considers the theory of Tversky and Kahneman (1991) as a special case when the beliefs assign all the probability to one particular product. By considering beliefs or expectations as the reference point it is possible to explain previous facts that seem to contradict the original findings of R-D preferences, for example why agents that expect to trade do not suffer from the endowment effect.

Most of the evidence that supports that agents have reference dependent preferences can be found in randomized controlled experiments⁶. This literature started with simple mug exchanges and developed later into more complex environments like sealed-bid auctions. The classical mug exchange experiment can be found in Kahneman et al (1990), where concepts such as endowment effect and status quo bias were tested for the first time. In the mug exchange experiments half of the subjects are randomly endowed with a good (usually mugs) to be kept or traded with the rest of the subjects. Neoclassical theory posits that half of the goods should be traded, however in most of these papers the authors show that only a few goods change hands (less than 10 percent of the mugs in Kahneman et al (1990)). In this kind of experiments the endowment acts as the reference point. The presence of loss aversion implies that to give up this endowment additional compensation is required, and so the willingness to pay is less than the willingness to accept. Testing for R-D preferences when the endowment is the reference point has been also documented in works of Isoni et al (2008) and in a field experiment

⁶The experimental literature regarding this topic is enormous and I will only mention a few papers.

by List (2003)

Recent experimental research tests the KR model by exogenously modifying agents' expectations. For example Yu et al (2017) varied the information shared with consumers regarding the waiting time in a call center. By structurally estimating an optimal stop model the authors show that the waiting cost increases once the real waiting time surpasses the expected one. In the same vein the experiment in Abeler et al (2011) manipulated the expected payment subjects could earn in an effort choice experiment. In their experiment subjects decided how much time to work on a tedious task. Once the subjects decided to stop an observed lottery assigned them to a payment condition. In the first condition subjects were paid a previously announced fixed rate, in the second condition subjects payment depended on how much time they worked. The authors show that as the probability of being paid in a fixed rate increases the subjects decide to work significantly less.

Using expectations as reference points but in the context of auctions Banerji and Gupta (2014) show how loss aversion changes bidding behavior. In their experiment students submit a sealed bid that plays against a random bid drawn from a known uniform distribution. If the student bid is greater than the number drawn by a computer, the student wins the auction but pays the random number. As in any second price auction neoclassical rationality implies that agents should submit their valuations. However the authors show in their model that loss aversion implies some shading of it. This is verified by observing how agents change their bids when the observed expected bid made by the computer varies.

Outside the experimental framework the complications of testing reference dependence magnifies, mostly due to the lack of information about the reference point. Camerer et al (1997) finds negative supply elasticities for taxi drivers in New York city. The authors suggests that cab drivers work as if they have target or reference income. Working more after reaching the target becomes extremely costly and thus most of the cab drivers stop. By modeling target income inside a supply model Farber (2008) tests the reference dependent hypothesis in a structural way. The author concludes that the target income varies too much across days

to be consistent with a “status quo” kind of reference point, hence disregarding the previous work. Nevertheless Crawford and Meng (2011) revisit Farber’s work considering reference income as a set of beliefs that account for the probabilistic demand, i.e., as in the KR model. The authors successfully explain why target income varies across days while finding consistent evidence of R-D preferences.

The role of R-D preferences inside the study of markets has been limited mostly to theoretical models. Heidhues and Köszegi (2008) modifies the Salop (1979) model to introduce R-D preferences in the same way as in the KR model. The authors show that even if firms face different marginal costs, a focal price equilibrium arises under a wide variety of conditions. Intuitively, when a firm decides to charge a higher price than the one consumers expect, loss aversion implies losing a considerable amount of the market share. If on the other hand a firm decides to charge a lower price, the winnings in market share are small compared to the markup loss. In this case R-D preferences help to explain price stickiness, and moreover why different firms tend to choose a similar price. The focal price equilibrium however requires that the reference point comes from consumers rational expectations. Forming a reference point with rational expectations seems possible in industries where consumers constantly make purchasing decisions.

Instead of using expectations, Zhou (2011) considers the case where the reference point is a product in the industry. By considering a duopoly that competes à la Hotelling, the author shows that the unique equilibrium is when the reference firm plays a high\low price mixed strategy, and the non-reference firm uses a medium price. The mixed strategy equilibrium depends heavily on the duopoly assumption. Angelatos and He (2017) extend the model to an arbitrary number of firms by modeling competition in a spoke network. In this case the equilibrium price strategy depends on consumers’ intrinsic valuation of the product. For low valuations the reference product is priced lower than the competing products, on the other hand for high valuations the reference product is priced higher than the competition. Having a reference product in the industry might be the result of publicity or advertisement.

In empirical IO the role of reference dependence is still very new. There exists

some skepticism on the value added by modeling R-D preferences inside a flexible enough choice model such as BLP. This research sheds some light of this value and presents situations where BLP falls short. In marketing, on the other hand, several choice models have been considered to explain reference prices. Reference price models incorporate loss aversion only in prices and not in product characteristics⁷. The reference point is defined by the researcher as the the previous price or as the expectation of all available prices. For example Mazumdar and Papatla (1995) conduct a simple logit model to estimate the price loss aversion parameter with scanner data on detergent and margarine. In a attempt to add heterogeneity, Kopalle et al (2012) uses the nested logit approach with scanner data on Cola beverages. Erdem et al (2001) builds a random utility model with normal correlated errors across brands on ketchup, peanut butter and tuna. All these papers find evidence of reference prices, nevertheless none of them takes into consideration the classical endogeneity problem of prices and unobservables. As noted in Berry (1994) failing to account for endogeneity might result in wrong estimates of the price coefficients. Finally from the IO perspective there is not a good reason to limit the research of R-D preferences only to the price domain, The model I propose allow for endogeneity and the reference point contains both prices and multiple product characteristics.

II. The R-D model

This section will first develop a general framework to introduce R-D preferences into discrete choice demand models. In pursuing comparability with BLP-type models, it will be useful to impose similar assumptions on the utility functions and substitution patterns. Once the general non-parametric model is presented, I will show natural simplifications that will be practical in empirical applications. Each of these specific models will be accompanied by propositions where the classical model fails to explain possible substitution patterns.

Building in the tradition of Berry et al (1995, 2004) a consumer i in market t

⁷The early work of Hardy et al (1993) considers also a reference quality, yet due to the hardships of measuring quality, subsequent papers focused in prices only.

chooses a good j from $\mathcal{J}_t = \{0, 1, \dots, J_t\}$ available products. Each product j in market t is described by a triplet $(p_{jt}, x_{jt}, \xi_{jt})$, where $p_{jt} \in \mathbb{R}_+$ is the price of the product; $x_{jt} \in \mathbb{R}^k$ is a vector of k observable characteristics; and $\xi_{jt} \in \mathbb{R}$ summarizes all unobserved characteristics that are relevant to the consumer's utility. As is common in the industrial organization field, prices are expected to be correlated with ξ ⁸. In terms of notation it will be convenient to label the outside good for all markets $t \in T$ with $j = 0$. Also, for exposition simplicity, I will suppress the market index t and bring it back when necessary.

Consumers are identified by a triplet (z_i, ζ_i, G_i) , where ζ_i captures unobservable tastes; $z_i \in \mathbb{R}^m$ is a vector of observable characteristics; and G_i is the reference point. As in the KR model, the reference point will capture beliefs or expectations that consumers have over the product space. In particular let the reference point G_i , be a probability measure over the observable product space (p_j, x_j) . The indirect utility of consumer i that is obtained by choosing product j , is given by:

$$(1) \quad U(z_i, \zeta_i, x_j, p_j, \xi_j; G_i) = \int u(\zeta_i, x_j, p_j, \xi_j; x_r, p_r) dG_i(x_r, p_r)$$

where $u(s_i, \zeta_i, x_j, p_j, \xi_j; x_r, p_r)$ is the indirect utility for a fixed reference product r . In this way, the model is flexible enough to capture perceptions of loss and gains with respect to different available products, previously available products, or even unavailable but expected features. Agents derive utility from two main parts: the consumption of the good, and the comparison of their purchase with respect to the reference point. This is captured by the following assumption.

Assumption 1 (Separability): Conditional on a given reference product r , the indirect utility of subject i that purchases good j is given by

$$(2) \quad u(z_i, \zeta_i, x_j, p_j, \xi_j; x_r, p_r, \xi_r) = v(z_i, \zeta_i, x_j, p_j, \xi_j) + \eta(x_j, p_j; x_r, p_r)$$

⁸ ξ_{it} could be also correlated with all or some of the components of x_{ij} , see Gandhi and Houde (2016).

where the first term $v(z_i, \zeta_i, x_j, p_j, \xi_j)$ captures the intrinsic utility of consuming good j ; the second term $\eta(x_j, p_j; x_r, p_r)$ models the “gain-loss” utility with respect to a reference product (x_r, p_r) .

The separation made by assumption 1 not only brings tractability to the model, it also says that the effects of a purchase not only depend on the reference point, the consumer still enjoys the product’s own features in the usual way. Naturally if $\eta(x_j, p_j; x_r, p_r) = 0$ the model simplifies to a standard discrete choice model; and so the R-D model nests the standard models.

The next step is to allow for different perceptions of gains and losses. For example, when making purchase a consumer might experience a loss in price yet at the same time a gain in quality. Thus, as in the KR model and in accordance with Tversky and Khaneman (1992), each component of the “gain and loss” function should be evaluated separately to be considered as a gain or a loss. The following assumption establishes the different categories where the model allows consumers to have different perceptions.

Assumption 2 (Categorization): Conditional on a given reference product r , the gain and loss utility can be decomposed as:

$$(3) \quad \eta(x_j, p_j; x_r, p_r) = \eta_p(p_j; p_r) + \eta_x(x_j; x_r)$$

Note that assumption 2 implies that consumers evaluate prices, and observable characteristics separately and possibly in different ways. However this assumption does not restrict the possibility of evaluating some or all observable characteristics at once. The reason behind this comes from the differences between horizontal and vertical attributes. Note that from the researcher point of view, vertical characteristics’ values can be easily cataloged as losses or as gains. A higher price than the expected reference price is a loss. However, horizontal characteristics depend on the consumer’s ideal taste, and for that reason horizontal characteristics require special treatment. Without loss of generality it is possible to assume that all observed characteristics are horizontal, noting that vertical characteristics can be

modeled in a similar fashion to how prices will be modeled. With this considerations it is possible to establish loss aversion in the model.

Assumption 3 (Loss Aversion in Vertical Characteristics): The loss and gain function associated with vertical characteristics (prices for exposition simplicity) can be written as follows:

$$n_p(p_j; p_r) = \mu_p(p_r - p_j)$$

Where $\mu_p : \mathbb{R} \rightarrow \mathbb{R}$ satisfies the following conditions

- $\mu_p(x)$ is continuous for all x , and $\mu_p(0) = 0$
- $\mu_p(x)$ is weakly increasing
- If $y > x > 0$ then $\mu_p(y) + \mu_p(-y) < \mu_p(x) + \mu_p(-x)$

Assumption 3 says that individuals somehow weight the difference between the reference product's price. Negative values or losses have more weight than positive values or gains. That is, individuals are loss averse. Assumption 3 is a weaker version of the assumptions made on the KR model⁹.

Assumption 4 (Loss Aversion in Horizontal Characteristics): The loss and gain function associated with observable characteristics is given by:

$$\eta_x(x_j; x_r) = \mu_x(-d(x_j, x_r))$$

where $d(\cdot, \cdot) : \mathbb{R}^k \times \mathbb{R}^k \rightarrow \mathbb{R}$ is a distance function and $\mu_x(\cdot)$ satisfies all previous characteristics.

Assumption 4 says that consumers pay a utility cost when buying something different in observable characteristics than what their reference product is. Loss aversion here is extreme, as there are no additional gains with respect to the reference product. Assumptions 1-4 provide the structure of the non-parametric model.

The non-parametric model reproduces the results of proposition 1 in Közsegi and Rabin (2006), and thus the R-D model translates their theory to the discrete choice framework . In this context, market shares (or demands) can be obtained if we

⁹Note that in this model the choice set is not stochastic. It is not necessary to impose the rest of the assumptions presented in the KR model.

aggregate the individual behavior over the population distribution. That is, let $\mathbf{Y}_i = (z_i, \zeta_i, G_i)$, with distribution $P_{\mathbf{Y}}$, and let

$$A_j(x_j, p_j, \xi_j) = \{\mathbf{Y} : U(z_i, \zeta_i, x_j, p_j, \xi_j; G_i) > U(z_i, \zeta_i, x_n, p_n, \xi_n; G_i) \forall n \in J\},$$

then the market shares implied by the model are given for every $j \in \mathcal{J}_t$ as:

$$(4) \quad \sigma_j(p, x, \xi) = \int_{A_j(x_j, p_j, \xi_j)} P_{\mathbf{Y}}(d\mathbf{y})$$

While the general model seems too ambitious to take it to the data, there exist natural simplifications that will allow us to do so.

A. The common reference product case

The first case will simplify the model in order to accommodate situations where industries have a unique reference product. As mentioned above, there exist several industries that fall into this category, and thus highlight the importance of R-D preferences in the real world. To see how to model this case, consider the following two simplifications

Simplification 1 (Degenerate Measure): For $(x_j, p_j, \xi_j) \in \text{supp}(G)$, there exists a product (x_r, p_r, ξ_r) such that

$$G_i(x_r, p_r) = 1$$

Simplification 2 (Common Reference Point): for every i ,

$$G_i = G \text{ a.e.}$$

Simplification 1 imposes the restriction that the reference point must be a product in the product space. It still allows for situations heterogeneity among consumers. An example of this type of industries would be industries with two leading brands. On the other hand, simplification 2 imposes a common reference product. With

these simplifications, the indirect utility reduces as follows:

$$(5) \quad U(z_i, \zeta_i, x_j, p_j, \xi_j; x_r, p_r) = v(z_i, \zeta_i, x_j, p_j, \xi_j) + \mu_x(-d(x_j, x_r)) + \mu_p(p_r - p_j)$$

Even in this simpler form, the model can produce novel substitution patterns. Before proceeding to the formal proposition, consider the following simple but illustrative example.

Suppose a market has three products, labeled a , b and r . These products are described by two characteristics, x_1 and x_2 . For simplicity, suppose that half the population prefers products that have more of characteristic x_1 ; the other half of the population has reversed preferences. Also assume that product a has more of x_1 than product b , but b has more of x_2 than a . Let r 's characteristics be the average of those from a and b . Without reference dependence and same prices consumers will most likely select a and b in similar proportions, and they will almost never select r . Unsurprisingly, if we allow for reference dependence with respect to r , the market share of r will come from stealing loss averse consumers, taken equally from a and b . Now consider the case where r increases the amount of x_2 such that it gets closer to b . Without reference dependence the market share of a is expected to weakly increase; nevertheless, loss aversion with respect to the distance to r makes a less attractive than before. At the same time product b starts looking better and its market share should weakly increase. Note that this change cannot be produced by most standard choice models, including BLP. From this simple example it is natural to expect that changes in the reference product across markets (time) helps to identify the gain and loss components of the utility function of the R-D model.

Even though the previous example imposed strong assumptions on the tastes of the population, it turns out that the same logic works for a broad class of tastes and observable characteristics (ζ_i, z_i) . Consider the set

$$\tilde{A}(x_j, p_j, \xi_j) = \{\mathbf{Y}_i : v(z_i, \zeta_i, x_j, p_j, \xi_j) > v(z_i, \zeta_i, x_n, p_n, \xi_n) \forall n \in J\}$$

The set $\tilde{A}(x_j, p_j, \xi_j)$ informs which part of the population derives more indirect intrinsic utility from product j than from the rest of the products. In the same way let $\tilde{\sigma}(x_j, p_j, \xi_j) = \int_{\tilde{A}(x_j, p_j, \xi_j)} P_{\mathbf{Y}}(d\mathbf{y})$. Note that $\tilde{\sigma}(x_j, p_j, \xi_j)$ are the market shares that traditional models will generate (i.e. when reference dependence does not matter). The following assumption restricts tastes (ζ_i, z_i) in a way that is natural for most of the applied models within the differentiated product world.

Assumption 5 (Intrinsic Weak Substitution): $\tilde{\sigma}_j(p, x, \xi)$ is weakly decreasing in any component of x_n or p_n for all $n \notin \{0, j\}$.

Even when at first glance assumption 5 might seem restrictive, it is not. Assumption 5 says that the industry consists of products which have a non-trivial degree of substitution, that is to say, the demand of product j could be shifted by changes in the components of product n . Indeed, assumption 5 is used in Mcfadden's Logit (1974), Berry's Nested Logit (1994), the linear random coefficient model in BLP (1995, 2004), the panel random coefficient model in Nevo (2001), and in Berry and Haile (2014). With assumption 5 it is possible to establish the main contribution of the model with R-D preferences.

PROPOSITION 1: *If Assumptions 1-5 hold, and under Simplifications 1 and 2 There exist values of (p, x, ξ) and $x_r < x'_r$ with $d(x'_r, x_j) < d(x_r, x_j)$, such that $\Delta\sigma_j > 0$*

Since in standard models, $x_r < x'_r$ with $d(x'_r, x_j) < d(x_r, x_j)$ always imply $\Delta\tilde{\sigma}_j < 0$, proposition 1 motivates the importance of accounting for reference dependent preferences within the field of industrial organization. Without it, the evaluation of mergers could result in an opposite prediction than when included. The real challenge of the reference product case might be to argue which is the actual reference product, which I will discuss later on.

B. Second scenario, heterogeneity on the reference point

The second scenario removes simplification 2 but keeps simplification 1, and so allows for different reference products among the population. While this case relates

to a broader set of industries, it introduces additional concerns. First, the empiricist needs to design a mechanism that reveals which is the reference product for each consumer in the data. Second, having multiple reference products diminishes the importance of R-D preferences on the market structure, and therefore raises the question on when and how it matters ¹⁰.

Depending on the application, the first concern can be addressed by using the information on previous purchases made by the observed consumers. For example, it is natural to expect that a consumer will use her current car to evaluate gains and losses on her next car purchase. Using the previous purchase as the reference point or price can be found in the work of Hardy et al (1993). On the other hand, in markets where there exist a few leader brands and many followers, some statistics that account for the whole history of observed purchases (such as the mode) can help detect the reference product, Baucells et al (2011). As we should expect, heterogeneity in reference points requires more from the data in question, and so limits its application to its availability.

To answer to what extent having multiple reference products matters for the industry study consider the following example. Suppose that in a given industry there exist 4 products, two of which are reference products. Denote as before the non-reference ones as a and b , and the reference ones as r_1 and r_2 . Each product is defined again by two characteristics x_1 and x_2 . In terms of x_1 the products are ordered as follows: $a > r_1 > b > r_2$. The order reverses for x_2 . Half of the consumer population prefers x_1 and the other half prefers x_2 . Moreover suppose that half of the population that likes x_1 has r_1 as reference product, and the other half has r_2 . The same occurs for the part of the population that prefers x_2 . Table 1 summarizes this information.

Without reference dependence and with similar prices, consumers will select a and r_2 in similar proportions, and they will almost never select r_1 or b . With reference

¹⁰For example the market shares predictions from an industry that have a large enough number of reference products will be very close to those, as in BLP. Even if the market shares are similar, the model could still be used to test whether agents have reference dependent preferences.

TABLE 1—CONSUMER PREFERENCES

	Prefers products that have more of x_1	Prefers products that have more of x_2
Reference r_1	$\frac{1}{4}$	$\frac{1}{4}$
Reference r_2	$\frac{1}{4}$	$\frac{1}{4}$

dependence, the market shares from a will decrease due to loss averse consumers that have either r_1 or r_2 as their reference product. Similarly, the market share from r_2 will decrease due to loss averse consumers that have r_1 as their reference. Suppose that r_1 increases its amount of x_1 such that it gets closer to a . Without reference dependence, the market share of a should weakly go down, since it has a closer substitute. Nevertheless reference dependence implies that the market share of a should weakly increase, note however that this increase comes only from the $\frac{1}{4}$ of the population that has r_1 as their reference and prefers x_1 , and not from the $\frac{1}{4}$ of the population that likes x_1 but has the reference product b . Similarly the market share of b or r_2 could go down since now they are farther away from r_1 . While this example says that reference can produce contrary substitution patterns, it also suggests that as the number of reference points increases the model will produce similar patterns to those under the classical theory. The following proposition formalizes this idea

PROPOSITION 2: *If Assumptions 1-5 and Simplification 1 holds then there exist an $\bar{R} \in \mathbb{N}$ such that*

1) *If the number of different reference products is less than \bar{R} , then*

There exist values of (p, x, ξ) and $x_{r_n} < x'_{r_n}$ with $d(x'_{r_n}, x_j) < d(x_{r_n}, x_j)$, such that $\Delta\sigma_j > 0$

2) *If the number of different reference products is greater than \bar{R} , then no such values of (p, x, ξ) and $x_r < x'_r$ exist*

Proposition 2 says that the model could be testable in many situations, however it imposes the warning that the researcher might lose the ability to distinguish the R-D model from the standard modal as the number of different reference products

grows. Note that even if the model allows for heterogeneity in the reference points, the model does not allow for heterogeneity on the degree of dependence the individual has, which as noted by Bodur and Arora (2014) is important when targeting consumers.

C. Stochastic Reference Product

Before moving to the estimation section it is important to discuss what happens if simplification 1 is removed, that is, when the model allows for a stochastic reference product. Having a stochastic reference product has two main advantages. The first one consists in letting the individuals to have an ideal product that is not necessarily available in the market. An individual could have a reference price equal to the average one of all available products, and at the same time expect some basic features (any smartphone should have a camera and should be touch screen). The second advantage comes from the researcher's point of view, there is no need to preemptively take a stand on which one is the reference product-rather the researcher allows the data answer it.

While these advantages seem interesting, the implications on the market structure are less clear. All previous arguments to distinguish the model and test the theory rely on observing movements in the reference product(s), which in this case are not directly observable. Note however that the model still predicts unexpected substitution patterns. To see this just replace the reference product(s) for fictional ones in the previous the examples. We should expect that in data sets which span long periods, the standard of comparison changes, somewhat helping to test an R-D model against standard ones. Nevertheless additional research is required to formally state this intuition and thus leaves this question to a future agenda.

III. Estimation and Simulations

When taking the model to the data it is necessary to introduce a parametric version of it. In particular this section will discuss how to estimate a linear random coefficient model for the case of a common reference product among consumers

(the extension to the case with heterogeneous reference products is almost direct, and the details can be found in the appendix). Among possible parametric forms, I choose the linear form to make a direct comparison to the classical BLP model. Such comparisons will be later discussed in this section through the use of simulated data. Specifically, I show how each model performs when either the true data generating process has no reference dependence, and when it actually does. The following assumption provides such desired structure to the model.

Assumption 6 (Linear Parametric Model): For all markets t and all products $j \in \mathcal{J}_t \setminus \{0\}$ the indirect utility of consumer i is given as follows:

$$(6) \quad u(z_i, \zeta_i, x_{jt}, p_{jt}, \xi_{jt}; \theta) = \sum_k x_{jkt} \tilde{\beta}_{ik} - \tilde{\alpha}_i p_{jt} + \xi_{jt} + \epsilon_{ijt} - \lambda(p_{jt} - p_{rt}) \mathbb{I}\{p_{jt} > p_{rt}\} - \gamma \|x_{jt} - x_{rt}\|_2$$

with

$$(7) \quad \begin{aligned} \tilde{\beta}_{ik} &= \bar{\beta}_k + \sum_r z_{ir} \beta_{kr}^o + \beta_k^u \nu_{ik} \\ \tilde{\alpha}_i &= \bar{\alpha} + \sum_r z_{ir} \alpha_r^o + \alpha^u \nu_{ip} \end{aligned}$$

and for all markets t , the utility of the outside good is $u(z_i, \zeta_i, x_{0t}, p_{0t}, \xi_{0t}) = \epsilon_{i0t}$ where $\nu_i = (\nu_{i1}, \dots, \nu_{ik}, \nu_{ip})$ is a random vector that models the unobserved individual heterogeneity for each observable characteristic and price, $\epsilon_{it} = (\epsilon_{i0t}, \epsilon_{i1t}, \dots, \epsilon_{iJ_t})$ is a mean zero vector of individual idiosyncratic shocks, and $\|\cdot\|_2$ is the L^2 -Norm.

In this particular model specification, the λ coefficient measures the consumers' loss aversion when purchasing a product that is more expensive than the reference product; in a similar way γ measures the loss aversion regarding the horizontal attributes. As in any parametric discrete choice model, an assumption over the consumers' unobservable features is required to predict market shares, and thus to estimate the model.

Assumption 7 (Tastes Distributions): In any market t , consumer i 's unobservables have the following underlying distributions:

- 1) ν_i is distributed joint normal with mean zero and identity covariance matrix
- 2) ϵ_{it} is distributed type-1 extreme value

The assumption 7 distributional forms are not a strict requirement, in fact ϵ_{it} could be replaced with a joint normal distribution, however assuming type-1 extreme value brings two advantages: first, it has a closed form that assists the exposition of the paper, second, it imposes less computational burden, and in fact the market shares can be written as follows:

$$(8) \quad \sigma_{jt} = \int \frac{\exp\left(\sum_k x_{jtk} \tilde{\beta}_i - \alpha p_{jt} + \xi_{jt} - \lambda(p_{jt} - p_{rt}) \mathbb{I}\{p_{jt} > p_{rt}\} - \gamma \|x_{jt} - x_{rt}\|_2\right)}{1 + \sum_{n=1}^{Jt} \exp\left(\sum_k x_{ntk} \tilde{\beta}_i - \alpha p_{nt} + \xi_{nt} - \lambda(p_{nt} - p_{rt}) \mathbb{I}\{p_{nt} > p_{rt}\} - \gamma \|x_{nt} - x_{rt}\|_2\right)} dF(\nu) d\bar{F}(z)$$

where $\bar{F}(z)$ and $F(\nu)$ are the distribution of observable characteristics and the distribution of unobservables. In practice the integral with respect to the unobservable characteristics is evaluated by Monte Carlo simulations by taking M draws from the distribution $F(\nu)$.

Before estimating the parameters $\theta = (\tilde{\beta}, \tilde{\alpha}, \lambda, \gamma)$, we need to address the endogeneity in prices that comes from the fact that firms observe ξ . Following the same ideas presented in Berry et al (1995, 2004), let w_{jt} be a vector of instruments that are correlated with the price, but uncorrelated with the unobservables ξ ; that is, when such instruments satisfy $E[\xi_{jt}|w_{jt}, x_{jt}] = 0$. This relationship allows us to form C moment conditions $E[\xi_{jt} \cdot h(w_{jt}, x_{jt})] = 0$ (where h are some known vector valued functions), and thus we can follow a GMM approach. To that end, let s_{jt} be the observed market share of product j in market t , and let $s_t = (s_{jt})_{j=0}^J$ be the collection of all market shares for each market. Since the implied market shares from the model have to match the observed ones ($\sigma_j(p, x, \xi; \theta) = S_j$), it is possible to use the MPEC approach to minimize a GMM criterion function. That is, we can follow the algorithm proposed in Dubé et al (2012), and hence write the estimation problem as:

$$(9) \quad \begin{aligned} & \min_{\theta, \xi} g(\xi)' \Omega g(\xi) \\ & \text{subject to } \sigma(p, x, \xi; \theta) = S \end{aligned}$$

where $g(\xi) = \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^J \xi_{jt} h(z_{jt}, x_{jt})$ and Ω is a GMM weighting matrix.

A. Evidence from Simulations

The following Monte Carlo simulations are intended to show the performance of the linear random coefficients R-D model vs the linear BLP model under different data generating processes. For that purpose, I consider a market structure based on $T = 30$ markets with $J = 10$ products. Each product is defined as the combination of $K = 3$ observable characteristics, its price p_{jt} and the unobservable ξ_{jt} . The simulated data comes from random draws of the following distributions:

$$\begin{pmatrix} x_j^1 \\ x_j^2 \\ x_j^3 \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & -0.8 & 0.3 \\ -0.8 & 1 & 0.3 \\ 0.3 & 0.3 & 1 \end{pmatrix} \right)$$

$$\xi_{jt} \sim i.i.d. N(0, 1), \text{ and } p_{jt} = |0.5\xi_{jt} + e_{jt} + x_{1j} + x_{2j}|$$

where $e_{jt} \sim N(0, 1)$. For each product j in market t , I generate a vector of instruments w_{jt} of dimension $D = 6$. Each component of this vector of instruments is obtained as follows, $w_{jtd} = u + \frac{1}{4}(e_{jt} + 1.1 \sum_{k=1}^3 x_{kjt})$, where $u \sim Uniform(0, 1)$. In addition, the following polynomial functions are generated $z_{jtd}^2, z_{jtd}^3, x_{jk}^2, x_{jk}^3, \prod_{d=1}^6 z_{jtd}, \prod_{k=1}^3 x_{jk}, z_{jtd}x_{j1}$ and $z_{jtd}x_{j2}$.

With the same data, I generate the observed market shares in two scenarios. The first scenario is a world without R-D preferences, where the true utility function of the individual i is given by equation (6) with the following parameters: $\bar{\beta} = (1 \ 1.5 \ .5)$, $\beta^u = (.70 \ .70 \ .70)$, $\bar{\alpha} = -3$, $\alpha^u = .44$, $\lambda = 0$ and $\gamma = 0$. In total there exist 6 parameters to estimate with 42 instruments.

Table 2 contains the results from estimating several models: The logit model, the BLP, the R-D logit and, the full R-D model. As expected the BLP model does a great job at estimating the parameters, nevertheless the differences between the BLP model and the full R-D model are statistically negligible. This result might not be surprising since the BLP model is nested in the R-D model. In fact, given the parametric structure imposed in the gain and loss function for the price comparison, it is possible to construct a hypothesis test that rules out the presence of a common reference product in the data generating process.

To see this in each of the t markets re-index the products in an ascending way with respect to their own price, call this new index $\tau \in [1, \dots, J_t]$. With this new indexation it is possible to write the mean utilities for each product τ as

$$\delta_\tau = x_\tau \beta - \alpha p_\tau \mathbb{I}\{\tau \leq r\} - \phi p_\tau \mathbb{I}\{\tau > r\} - \gamma d(x_\tau, x_r) + \lambda p_r \mathbb{I}\{\tau > r\} + \xi_r$$

where $\phi = \alpha + \lambda$. This setting allow us to use the findings from the structural break literature, in particular adapting the ideas in Andrews (2003). Let $\varsigma = [\tau_1, \dots, \tau_{J_t}]$ where $\tau_1 > 1$ and $\tau_2 < J_t$. Let Ψ_r be the Wald Statistic for testing

$$H_0 : \begin{pmatrix} \alpha - \phi \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

when the possible reference product is r . Then it is possible to construct the supremum statistic $Sup\Psi = \sup_{\tau \in \varsigma} \Psi_\tau$, and test the null hypothesis of a data set without a common reference product using the critical values provided by Andrews (2003). The last line of Table 2 presents the value of the relevant information of this hypothesis where, as shown, I cannot reject the null. In other words, there is no evidence of a reference product in the data.

The second simulation contains preferences that are actually reference dependent. In particular, the parameters associated with the gain and loss functions take the following values $\lambda = -5$ and $\gamma = -4$. The results of estimating the same models as before are presented in Table 3. As expected the full R-D model does the best

TABLE 2—THE TRUE DGP HAS NO REFERENCE DEPENDENCE

	True Value	Logit	BLP	R-D Logit	R-D
$\bar{\alpha}$	-3.0	-4.33 (0.2182)	-3.11 (0.0082)	-2.24 (0.0153)	-3.10 (0.0080)
α^u	0.44	-	0.48 (0.0088)		0.475 (0.0024)
$\bar{\beta}_1$	1.0	0.65 (0.0345)	0.91 (0.0240)	1.64 (0.0023)	0.945 (0.0243)
β_1^u	0.70	-	0.44 (0.0153)		0.49 (0.0139)
$\bar{\beta}_2$	1.5	1.13 (0.0003)	1.31 (0.0235)	2.1 (0.0011)	1.30 (0.0238)
β_2^u	0.70	-	0.91 (0.0099)		0.97 (0.0093)
$\bar{\beta}_3$.50	0.66 (0.1243)	0.51 (0.0156)	0.68 (0.0141)	0.50 (0.0158)
β_3^u	0.70	-	0.48 (0.0088)		0.71 (0.0086)
λ		-		0.58 (1.311)	0.06 (0.0067)
γ		-		0.00 (1.22)	-0.03 (0.0013)
sup Ψ	12	<	Critical Value	27.53	Cannot Reject H_o

Note: Standard Errors are reported in brackets. The results come from evaluating the integral 1000 times. Several initial values were provided to MatLab with Knitro.

Source: Simulated data

job at estimating the parameters, however now the results from BLP and Logit are way off for most of coefficients. In particular we can see that the mean price sensitivity is overestimated. This is not a particular result of the simulation, but rather an implied result of the parametric model. Note that when estimating the BLP model, the price coefficient adds together its own sensitivity coefficient and the coefficient of loss aversion for those products that have a higher price than the reference. This is particularly important if demand estimates are inputs to additional models, such as merger evaluations or price policy counterfactuals. Note that the loss aversion coefficients are significant, implying that we reject the null hypothesis of non-reference dependence regarding that product.

As mentioned in the previous section, predictions regarding changes in market

TABLE 3—THE TRUE DGP HAS REFERENCE DEPENDENCE

	True Value	Logit	BLP	Logit-R-D	R-D
$\bar{\alpha}$	-3.0	-13.71 (0.1233)	-18.17 (0.0713)	-2.83 (0.0010)	-3.09 (0.0006)
α^u	0.44	-	3.36 (0.0180)		0.47 (0.0002)
$\bar{\beta}_1$	1.0	6.36 (1.221)	1.92 (0.1051)	1.60 (0.0415)	0.92 (0.0018)
β_1^u	0.70	-	0.00 (0.3634)		0.477 (0.0010)
$\bar{\beta}_2$	1.5	13.85 (1.001)	8.97 (0.0541)	2.10 (0.1457)	1.28 (0.0017)
β_2^u	0.701	-	0.00 (0.0934)		0.94 (0.0007)
$\bar{\beta}_3$.50	-0.85 (0.0124)	-1.30 (0.0735)	0.64 (0.0472)	0.49 (0.0012)
β_3^u	0.701	-	0.17 (0.0933)		0.67 (0.0006)
λ	-5.0	-	-	-4.45 (0.0001)	-4.92 (0.0005)
γ	-4.0	-	-	-3.99 (0.0011)	-4.16 (0.0002)

Note: Standard Errors are reported in brackets. The results come from evaluating the integral 1000 times. Several initial values were provided to MatLab with Knitro.
Source: Simulated data

shares could go in opposite directions between the models that include R-D preferences and traditional ones. Table 4 presents such a scenario in my simulation. In detail, I exogenously changed the characteristics of the reference product to make it identical to a competing product j . As shown, the market share of this product went up in reality, and the prediction of the R-D model captures it. However BLP wrongly predicts that given this change the market share should go down.

IV. Empirical Application: Data Description

The remaining sections of the paper discuss an empirical application of the R-D model. Since the main goal is to show that R-D preferences matter when estimating demand, I focus on an industry with a common reference product—the facial tissue industry. To that end, I use retail scanner data from IRI academic

TABLE 4—PREDICTIONS IN MARKET SHARES CHANGES

Before		After		
$d(x_j, x_r) = 1.76$		$d(x_j, x_r) = 0$		
	True Value	True Value	Prediction BLP	Prediction R-D
σ_j	0.06	0.24	0.02	0.30

Note: The results come from evaluating the integral 1000 times. Several initial values were provided to MatLab with Knitro.

Source: Simulated data

dataset households panel. This panel follows consumers in two U.S. cities, Eu Claire, Wisconsin and Pittsfield, Massachusetts. The information was recorded weekly, starting in January 2001 and ending in December 2012. Besides recording purchasing decisions, the panel contains household demographics such as income, education, family size, and race. Table 5 presents demographics for the whole panel. On average, each region is similar in terms of income and percentage of white households. However, some differences exist in the number of education years in the household as well as family size.

TABLE 5—AVERAGE DEMOGRAPHICS

	2001	2006	2012
Eu Claire			
Income	\$35,000 to \$44,999	\$35,000 to \$44,999	\$35,000 to \$44,999
White	97 Percent	98 Percent	98 Percent
Education	12 Years	12 Years	13 Years
Family Size	3 Persons	2 Persons	3 Persons
Households	2,117	2,192	1,366
Pittsfield			
Income	\$35,000 to \$44,999	\$35,000 to \$44,999	\$45,000 to \$54,999
White	94 Percent	98 Percent	96 Percent
Education	12 years	14 years	15 Years
Family Size	3 Persons	3 Persons	4 Persons
Households	2,908	2,020	1,228

Note: Income is the combined family yearly income before taxes.

Source: IRI Academic Data Set.

The main goal of using micro-data is to allow for observed heterogeneity among households. The following histogram shows the spread of income brackets in the

year 2006. As desired, each bracket has a considerable amount of density. Moreover the distribution shape resembles a typical income distribution for the United States population.

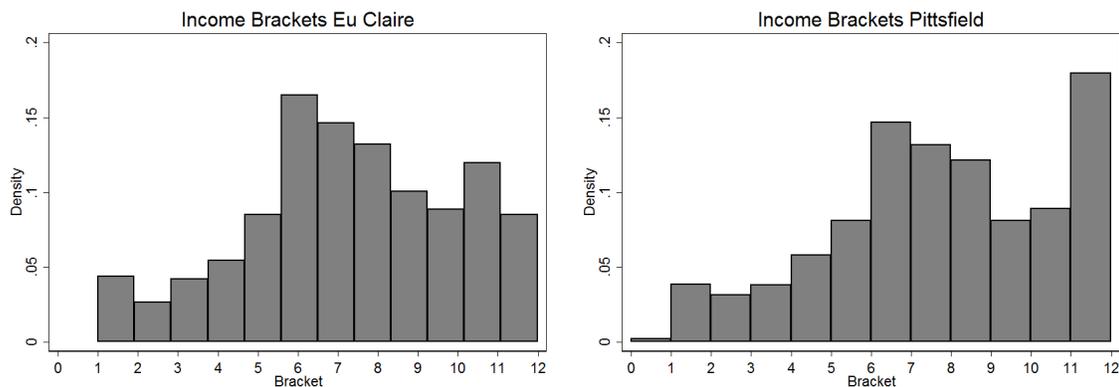


FIGURE 1. INCOME BRACKETS FOR 2006

Note: The income brackets are defined as follows: 1=0 to \$ 9,999; 2 = \$10,000 to \$11,999; 3 = \$12,000 to \$14,999; 4 = \$15,000 to \$19,999; 5 = \$20,000 to \$24,999; 6 = \$25,000 to \$34,999; 7 = \$35,000 to \$44,999; 8 = \$45,000 to \$54,999; 9 = \$55,000 to \$64,999; 10 = \$65,000 to \$74,999; 11 = \$75,000 to \$99,999; 12 = \$100,000 and greater.

Source: IRI Academic Data Set

A. *The Facial Tissue Industry*

The western facial tissue industry started when a company in Neenah, Wisconsin introduced a disposable tissue in 1924, known as Kleenex. Originally tissues were meant to remove makeup and were not substitutes for the common handkerchief. Around that time some research suggested that the common handkerchief transmitted diseases, including the cold virus. By advertising this fact, Kleenex became the perfect safe substitute of the handkerchief. The company was bought by Kimberly-Clark in 1955, and since then they have been the market leaders in the industry. The brand Kleenex became so popular in western countries that the word Kleenex is accepted as a synonym for facial tissue¹¹. Therefore Kleenex is

¹¹For example, The Royal Academy of the Spanish Language accepts Clínex (which is how Kleenex sounds like in Spanish) as a correct word for facial tissue.

a perfect candidate for a reference product.

During the period studied in this research, facial tissue industry composition contained a small number of firms that produced a large number of differentiated products. Table 6 presents the number of products offered by firms over time. As shown, this number changes across years and cities, however Kimberly-Clark dominates the industry in terms of number of products. In a given store, each firm

TABLE 6—NUMBER OF PRODUCTS

	Pittsfield			Eu Claire		
	2001	2006	2012	2001	2006	2012
Kimberly-Clark (Kleenex)	19	18	22	21	24	21
Procter and Gamble (Puffs)	14	17	14	14	22	13
Irving Tissue Converters (Scotties)	9	9	11	7	5	5
Private Label	11	14	22	8	14	13
Other	6	7	10	7	8	4
Total	59	65	79	57	73	56

Note: The Other category contains combined information of firms with less than 1 percent of market share. Source: IRI academic dataset.

only produces one brand. Figure 2 shows the market shares by top brands over time. As seen in the graph, Kleenex, on average, has more than 40 percent of the market, and in Eau Claire more than 60 percent in the first years. In early years, the brand Puffs was the closest competitor to Kleenex, whereas in later years the brands produced in retail stores became more popular. Scotties has consistently come in third place, and less than 1 percent of the market is divided among other brands.

Earlier in the discussion I suggested Kleenex as a reference brand. Since the reference product acts as a standard for comparison, it would be natural to observe that Kleenex as a brand is mentioned often when consumers review products. Unfortunately the IRI academic dataset does not ask households to review products. As a proxy for the missing reviews, Table 7 presents online reviews made in Amazon and Walgreen's. Note that in the Amazon's case no private label exists. Walgreen's, on the other hand, sells Nice! but does not sell Scotties. In each category Kleenex is

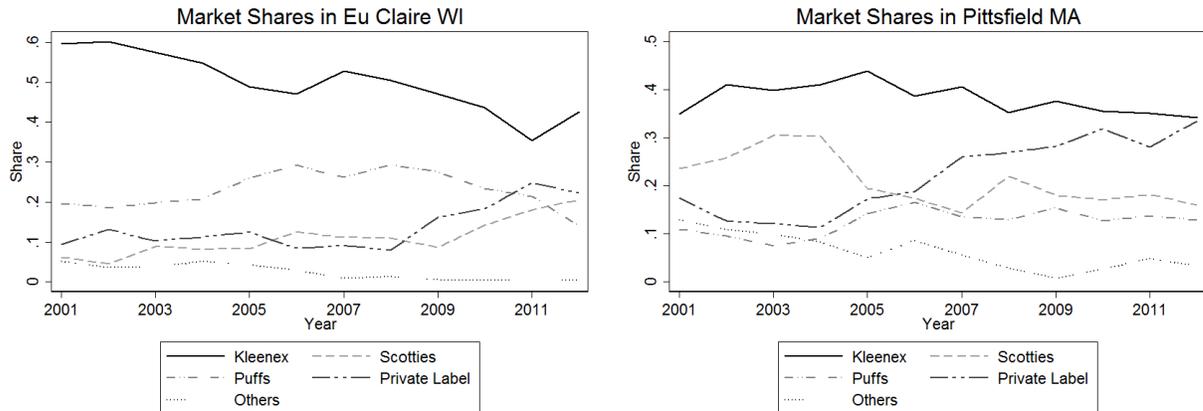


FIGURE 2. MARKET SHARES ACROSS TIME

Note: The Other category contains combined information of firms with less than 1 percent of the market share.

Source: IRI academic dataset.

mentioned more often than competition, supporting the idea of a reference brand.

TABLE 7—ONLINE REVIEWS

	The Brand Mentions Kleenex	Kleenex Mentions the Brand
Amazon		
Puffs	5 percent out of 639 reviews	.1 Percent out of 1,225 reviews
Scotties	2 percent out of 62 reviews	0 Percent of 1,225 reviews
Nice!	Unavailable	Unavailable
Walgreen's		
Puffs	7 percent out of 1186 reviews	2 Percent out of 823 reviews
Scotties	unavailable	unavailable
Nice!	20 percent out of 23 reviews	0 Percent out of 823 reviews

Note: The reviews were obtained as the time of writing the paper, and I consider the product that has more reviews.

Each brand produces different products. For the purpose of the model, a product is defined by a combination of observable characteristics, brand, and unobservables. Table 8 summarizes the available observable characteristics in the dataset, and also shows how often a product with that characteristics has been purchased. As expected the most popular characteristics are the rectangular box and the color white for the tissues. On average the number of sheets is 166, although considerable

dispersion exists. As noted in previous research, price variation is essential when

TABLE 8—SHARES OF PRODUCT CHARACTERISTICS

	Average on All Markets	Standard Deviation
White	0.87	0.3383
2 Plies	0.83	0.3329
Top Dispenser	0.81	0.3927
# of Sheets	166.40	104.9780
Aloe, Lotion or Antiviral	0.12	0.3310
Rectangular Box	0.92	0.2592
Cubic Box	0.07	0.2504
Plastic Wrap	0.01	0.0718

Note: All variables are dummy variables, except the number of sheets in the product
 Source: IRI Academic Data Set.

estimating demand systems. Figure 3 shows how prices vary in my data. This observed variation was obtained by defining the market as a quarter of a year and a city. This is the same market specification used in Nevo (2001), and will be used for estimating the model.

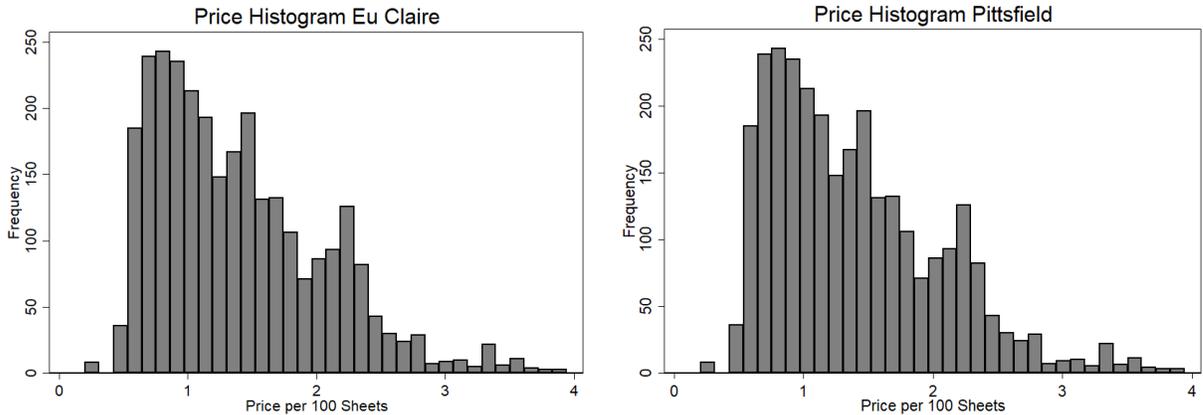


FIGURE 3. PRICE VARIATION

Note: Quarterly observed prices, from January 2001 to December 2012.
 Source: IRI Academic Dataset.

So far I have considered Kleenex as the reference brand, however it is necessary to choose which Kleenex product is the reference one. One possibility is to choose

the basic product that the company offers. The other possibility is to choose the most popular. Fortunately in this case the basic product coincides with the most popular one. This product is the rectangular Kleenex box with a top dispenser which contains plain white tissues formed by 2 plies. This product has been offered containing different number of sheets. It contained 160 sheets from the years 2001-2005 (65 percent of the all sizes were 160). The company decided to eliminate the 160 sheets in 2006 and introduce a product with 200 sheets instead. As seen in the identification section, such changes allows me to identify the R-D model. Before estimating the model, I present in Table 9 the correlations between the market shares and the distance with respect to the reference product. Although we cannot asses a causal direction, it shows evidence that being far from the reference product is correlated with having lower market shares.

TABLE 9—CORRELATION BETWEEN MARKET SHARES AND THE REFERENCE PRODUCT DISTANCE

	All Brands	Excluding Kleenex
Correlation	-0.3153	-0.2277

Note: The distance is the Euclidean distance with respect to observable characteristics of the reference product.

Source: IRI Academic Data Set.

V. Empirical Application: Results

The results presented in this section come from estimating several demand models across 2 cities and 48 quarters (such that, in total, the model considers 98 markets). Overall 4,833 inside products exist, with only 5 brands. As suggested by Nevo (2001), brand dummies are included for all models. Each of these models use BLP instruments to account for the endogeneity problem of prices.¹²

Table 10 starts the discussion by presenting the Logit results. Even when the Logit models capture unrealistic substitution patterns, we can still see the previously derived insights from the R-D model. Both OLS and Logit models estimate a

¹²The instruments on BLP are the sum over the characteristics produced by the firm, and the sum on the characteristics produced by the competition.

larger price coefficient (in absolute terms) than their R-D counterparts. Even when on average the own price elasticities are similar for both type of models (1.71 IV and 1.75 IV Logit), their predictions drastically differ with respect to the reference product price position. In particular, the average elasticity for those products that are less expensive than the reference is -0.33 for the R-D Logit, and -0.70 for the IV Logit model. For products that are more expensive, the predicted elasticities are -2.10 and -1.97.

TABLE 10—LOGIT RESULTS

	OLS	OLS R-D	IV	IV R-D
Price per 100 sheets	-0.5381 (0.0212)	-0.2530 (0.056)	-1.022 (0.0531)	-0.5043 (0.2972)
Size	-0.0495 (0.0124)	-0.0218 (0.003)	-0.1396 (0.0954)	-0.1503 (0.001)
White	-0.3814 (0.1680)	-0.1362 (0.0785)	0.1396 (0.0954)	0.1025 (0.1022)
2 Plies	0.2056 (0.0549)	0.3029 (0.0563)	0.4479 (0.0642)	0.3899 (0.0715)
Top Dispenser	0.3985 (0.0586)	0.4301 (0.0604)	0.4128 (0.0627)	0.3496 (0.0687)
Aloe	-0.3991 (0.0591)	-0.2562 (0.0624)	-0.3350 (0.0654)	-0.2908 (0.0710)
Cubic Box	0.3883 (0.1010)	0.8650 (0.0980)	0.3696 (0.1251)	0.3550 (0.1447)
Rectangular Box	0.6200 (0.0841)	0.8419 (0.0797)	0.3844 (0.1058)	0.3053 (0.1247)
Loss Aversion Price (λ)		-0.76648 (14.250)		-0.5878 (0.3670)
Loss Aversion Characteristic Space (γ)		-0.2586 (0.0460)		-0.1088 (0.0497)

Note: BLP instruments. Brand dummy variables included.
Source: IRI academic dataset.

The estimated values for the full R-D model and the BLP model are presented in the following two tables. Table 11 contains the results for the mean and random coefficients. At first glance, it is possible to see that most of the mean utilities have

the same direction and similar magnitude in both models. However, as in the Logit model, the mean price coefficient ($\bar{\alpha}$) is larger in BLP. This fact is also present in the estimated values of the random coefficient α^u . Since the R-D model has an additional degree of freedom to accommodate changes in prices, it is conceivable to expect that BLP will rely on greater unobserved heterogeneity to accommodate the same data. Note that the coefficient which captures loss aversion with respect to the distance is significant and its magnitude is comparable to the the rest of the characteristics magnitude. Even when the reference product comes in a rectangular box, premiums exist for the cubic box. The last line of Table 11 presents the value of the structural break hypothesis. As expected, the data rejects the null of a common reference product absence.

Table 12 contains the results for the interaction terms with micro-data. I tried several specifications of which variables to include; here I am only reporting those that yielded significant results. Roughly the same findings are maintained in both models, noting that higher income is correlated with less disutility on prices. Wealthier families also tend to prefer tissues with add-ons such aloe, and more plies. Family size has a negative relationship with respect to the size and prices.

So far, I have shown evidence that R-D preference matters for the study of the facial tissue industry. Now I plan to show evidence for how much it does. To that end, I predict what the changes in market shares would be if consumers did not have reference dependent utility, that is if $\mu = 0 \gamma = 0$. Table 13 presents the percentage change of a subset of products. Note that the reference product loses 25% of its market share, but it is also the case that close competing products lose market share. The biggest loser is the identical private label, which before was gaining market share by offering a similar product to the reference, but cheaper. Puffs, on the other hand, seems to enjoy this change.

VI. Discussion

This research has presented a model that allows the empirical economist to estimate discrete choice models of demand with reference dependent utility.

TABLE 11—RANDOM COEFFICIENT MODELS

	Mean		Deviation	
	Parameters		Parameters	
	R-D	BLP	R-D	BLP
Price per 100 sheets	-1.4380 (0.4720)	-3.8512 (0.6891)	2.531 (1.987)	7.152 (2.187)
Price Loss Aversion (λ)	-1.07 (0.1412)	- -	- -	- -
White	-0.1201 (0.0004)	-0.2201 (0.0010)	0.278 (0.003)	0.3045 (0.0954)
2 Plies	0.3210 (0.1336)	0.8580 (0.0729)	0.132 (0.0126)	0.1868 (0.0234)
Top Dispenser	0.4256 (0.1853)	0.5560 (0.0921)	2.142 (0.5013)	1.1616 (0.0159)
Size	-0.2514 (0.0011)	-0.1719 (0.3067)	1.62 (0.0520)	0.2009 (0.4847)
Aloe	-0.3727 (0.211)	-0.2227 (0.2127)	3.195 (0.0150)	1.3505 (0.0319)
Cubic Box	1.324 (0.2142)	0.461 (0.0273)	0.199 (0.0219)	1.7392 (0.7370)
Rectangular Box	0.818 (0.0285)	0.9009 (0.0095)	0.2812 (0.0376)	0.7422 (0.9952)
Loss Aversion (μ)	-0.3520 (0.0151)	- -	- -	- -
Sup Ψ	55	>	Critical Value	14.23

Note: BLP instruments. Brand dummy variables included. Integral simulated with 1000 draws. Tests based on critical values provided by Andrews (2003).

Source: IRI academic dataset.

Accounting for reference dependent utility is extremely relevant in industries where a common reference product exists among consumers. In these industries, ignoring R-D preferences can bias the estimation results in two ways. First, the estimates may exaggerate how consumers react to prices. This phenomenon occurs because standard models will confound the price sensitivity with loss aversion. Identifying these effects separately is important when trying to model firm's price decisions, or when conducting policy counterfactuals. Second, and perhaps most important, standard models assume that market shares are hindered when facing close substitutes. On the other hand, the R-D model permits competing products to enjoy benefits (losses) for being closer (further away) to the reference product.

TABLE 12—MICRODATA INTERACTION TERMS

	Interaction Income		Interaction Family Size		Interaction Education	
	R-D	BLP	R-D	BLP	R-D	BLP
Price per 100 sheets	22.050 (3.5212)	15.050 (2.1215)	-0.851 (0.0121)	-0.851 (0.1014)	-	-
White	-	-	-0.5214 (0.021)	-0.3321 (0.0156)	-	-
2 Plies	-0.002 (0.0014)	-0.001 (0.0016)	-	-	-	-
Top Dispenser	-	-	-	-	-0.2832 (0.0142)	0.5321 (0.0425)
Size	-0.21 (0.1210)	-0.15 (0.1210)	2.55 (0.7251)	1.07 (0.5151)	-	-
Aloe	1.00 (0.0663)	1.25 (0.3343)	-	-	-0.112 (0.0043)	-0.1842 (0.5521)
Cubic Box	2.03 (1.1820)	2.03 (.9988)	-	-	-	-
Rectangular Box	-0.021 (0.0174)	-0.0093 (0.0186)	-	-	-	-

Note: BLP instruments. Brand dummy variables included. Tests based on critical values provided by Andrews (2003). Integral simulated with 1000 draws.
Source: IRI academic dataset.

This fact might lead the researcher to opposite conclusions when trying to predict changes in market shares. This is how the R-D model provides richer substitution patterns, which as shown in the identification results, can be tested.

An example of an industry with a common reference product is the facial tissue industry. A consumer that purchase a product that is different from the basic Kleenex product pays an additional utility cost. This cost provides Kleenex an advantage, and in fact, my estimates suggest that without it their market share would go down by 25 percent. Since R-D preferences benefit the reference product's close substitutes, competing products would be also affected if the loss and gain component disappeared. With respect to that, my estimates suggest that private labels that imitate Kleenex would lose market share as well (33 percent). Puffs, on the other hand, could have a larger market share if consumers were not reference

TABLE 13—COUNTERFACTUAL

	Change Market Share	Difference in Characteristics
Kleenex reference product	-25%	None
Kleenex closest competing product	-10%	Size
Kleenex farthest competitor	2%	All but brand
Puffs closer competitor	-5%	Brand
Puffs farthest competitor	35%	All
Scotties closer competitor	2%	Brand
Scotties farthest competitor	5%	All
Private Label closest competitor	-33%	Brand
Private Label farthest competitor	-2%	All

Note: Counterfactual assuming $\lambda = \gamma = 0$. Integral simulated with 1000 draws.
Source: IRI academic dataset.

dependent. These findings suggest that firms have incentives to advertise to position themselves as the reference product. In fact, accounting for R-D preferences when modeling supply decisions could be possible avenues for new applied research.

Most industries contain heterogeneity among consumers with respect to their reference product. While this research has pointed out that the R-D model still produces richer substitution patterns, the difference with respect to standard model becomes less important as the heterogeneity among consumers increases. In other words, we shouldn't be too worried on missing substitution patterns in industries like the automobile or the ready-to-eat cereal. However, interesting insights might result from estimating an R-D model in the soft drinks industry, or in the over-the-counter painkillers.

Finally, the general R-D model allows for the possibility of a stochastic reference product. While more research is needed to provide a formal identification argument, its an appealing model for several reasons. One of them is relaxing the burden on the researcher to specify the reference product in an industry. Second, it incorporates the well-documented notion that consumers form beliefs on prices with respect to previous purchases. Most likely a consumer that pays \$ 500 dollars for a domestic flight will experience a loss sensation if she had previously only booked cheaper flights.

The tools provided in this paper are first steps into building connections between the findings from behavioral economics and those from the structural study of markets. It opens a research agenda that should complement both fields.

REFERENCES

- Abeler, J., Falk, A., Goette, L., & Huffman, D. (2011). Reference points and effort provision. *The American Economic Review*, 101(2), 470-492.
- Amaldoss, W. & He, C. (2017) Reference-Dependent Utility, Product Variety and Price Competition. Working paper
- Banerji, A., & Gupta, N. (2014). Detection, identification, and estimation of loss aversion: Evidence from an auction experiment. *American Economic Journal: Microeconomics*, 6(1), 91-133.
- Baucells, M., Weber, M., & Welfens, F. (2011). Reference-point formation and updating. *Management Science*, 57(3), 506-519.
- Berry,
S. T. (1994). Estimating discrete-choice models of product differentiation. *The RAND Journal of Economics*, 242-262.
- Berry, S., Levinsohn, J., & Pakes, A. (1995). Automobile prices in market equilibrium. *Econometrica: Journal of the Econometric Society*, 841-890.
- Berry, S., Levinsohn, J., & Pakes, A. (2004). Differentiated products demand systems from a combination of micro and macro data: The new car market. *Journal of political Economy*, 112(1), 68-105.
- Camerer, C. F., Loewenstein, G., & Rabin, M. (2011). *Advances in behavioral economics*. Princeton University Press.
- Camerer, C. F., (1997) Labor supply of New York City cabdrivers: One day at a time. *The Quarterly Journal of Economics* 112(2), 407-441.
- Camerer, C. F., & Malmendier, U. (2017). *Behavioral Organizational Economics*. Yrjö Jahnsson Foundation conference on economic institutions and behavioral economics

- Crawford, V. P., & Meng, J. (2011). New York City cab drivers' labor supply revisited: Reference-dependent preferences with rational expectations targets for hours and income. *The American Economic Review*, 101(5), 1912-1932.
- DellaVigna, S. (2009). Psychology and economics: Evidence from the field. *Journal of Economic Literature*, 47(2), 315-372.
- Ellison, G. (2006). Bounded Rationality in Industrial Organization. Chapter. In *Advances in Economics and Econometrics: Theory and Applications Ninth World Congress*, edited by R. Blundell, W. Newey and T. Persson. 2, 142-97. *Econometric Society Monographs*. Cambridge: Cambridge University Press.
- Erdem, T., Mayhew, G., & Sun, B. (2001). Understanding reference-price shoppers: a within-and cross-category analysis. *Journal of Marketing Research*, 38(4), 445-457.
- Farber, H. S. (2008). Reference-dependent preferences and labor supply: The case of New York City taxi drivers. *The American Economic Review*, 98(3), 1069-1082
- Hardie, B. G., Johnson, E. J., & Fader, P. S. (1993). Modeling loss aversion and reference dependence effects on brand choice. *Marketing Science*, 12(4), 378-394.
- Hart, O. (1990). Is "Bounded Rationality" an Important Element of a Theory of Institutions?. *Journal of Institutional and Theoretical Economics*, 146(4), 696-702.
- Heidhues, P., & Köszegi, B. (2008). Competition and price variation when consumers are loss averse. *The American Economic Review*, 98(4), 1245-1268.
- Hendricks, K. (2006). Price Discrimination and Irrational Consumers: Discussion of Armstrong and Ellison. In *Advances in Economics and Econometrics: Theory and Applications Ninth World Congress*, edited by R. Blundell, W. Newey and T. Persson. 2, 175-81. *Econometric Society Monographs*. Cambridge: Cambridge University Press.
- Isoni, A., Loomes, G., & Sugden, R. (2008). The Willingness to Pay-Willingness to Accept Gap, the 'Endowment Effect', Subject Misconceptions, and Experimental Procedures for Eliciting Valuations.

- Kahneman, D. (2003). Maps of bounded rationality: Psychology for behavioral economics. *The American economic review*, 93(5), 1449-1475.
- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the econometric society*, 263-291.
- Kahneman, D., Knetsch, J. L., & Thaler, R. H. (1990). Experimental tests of the endowment effect and the Coase theorem. *Journal of political Economy*, 98(6), 1325-1348.
- Kopalle, P. K., Kannan, P. K., Boldt, L. B., & Arora, N. (2012). The impact of household level heterogeneity in reference price effects on optimal retailer pricing policies. *Journal of Retailing*, 88(1), 102-114.
- Kőszegi, B., & Rabin, M. (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics*, 121(4), 1133-1165.
- List, J. A. (2004). Neoclassical theory versus prospect theory: Evidence from the marketplace. *Econometrica*, 72(2), 615-625.
- Mazumdar, T., & Papatla, P. (1995). Loyalty differences in the use of internal and external reference prices. *Marketing Letters*, 6(2), 111-122.
- Mullainathan, S., & Thaler, R. H. (2000). Behavioral economics (No. w7948). National Bureau of Economic Research.
- Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *Econometrica*, 69(2), 307-342.
- Petrin, A. (2002). Quantifying the benefits of new products: The case of the minivan. *Journal of political Economy*, 110(4), 705-729
- Salop, S. C. (1979). Monopolistic competition with outside goods. *The Bell Journal of Economics*, 141-156.
- Spiegler, R. (2011). Bounded rationality and industrial organization. Oxford University Press.
- Tversky, A., & Kahneman, D. (1991). Loss aversion in riskless choice: A reference-dependent model. *The quarterly journal of economics*, 106(4), 1039-1061.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5(4), 297-323.

Yu, Qiuping and Allon, Gad and Bassamboo, Achal (2017). The Reference Effect of Delay Announcements: A Field Experiment, Kelley School of Business Research Paper No. 17-26.

PROOFS OF PROPOSITIONS

Proposition 1

PROOF:

Start by conditioning on (p, ξ) .

Rewrite the set A_j as follows:

$$A_j = \{w \in \mathbf{W} : U(w, x_j; x_r) > U(w, x_n; x_r) \forall n \in J\}$$

By using assumptions 1-4 and simplification 1-2 it becomes the following set:

$$A_j = \{w \in \mathbf{W} : v(w, x_j) + \mu(-d(x_j, x_r)) > v(w, x_n) + \mu(-d(x_n, x_r)) \forall n \in J\}$$

In a similar way write the set \tilde{A}_j as follows

$$\tilde{A}_j = \{w \in \mathbf{W} : v(w, x_j) > v(w, x_n) \forall n \in J\}$$

By assumption 5, when x_r increases to x'_r such that $d(x_j, x'_r) < d(x_j, x_r)$ we obtain that

$$\int_{\tilde{A}_j} P_{\mathbf{W}}(d\mathbf{w}) \geq \int_{\tilde{A}'_j} P_{\mathbf{W}}(d\mathbf{w})$$

where \tilde{A}'_j is the corresponding set when the new product space, J' , contains x'_r instead of x_r .

In other words given a realization of w ,

$$\text{Max}_{n \in J'} \{v(w, x_n)\} = v(w, x'_m) > v(w, x_m) = \text{Max}_{n \in J} \{v(w, x_n)\}$$

Take $w \in A_{j'} = \{v(w, x_j) + \mu(-d(x_j, x'_r)) > v(w, x'_m) + \mu(-d(x'_m, x'_r))\}$

By rearranging terms, it is easy to see that such w satisfies

$$\mu(-d(x_j, x'_r)) - \mu(-d(x'_m, x'_r)) > v(w, x'_m) - v(w, x_j)$$

Since $v(w, x'_m) > v(w, x_m)$, w also satisfies

$$\mu(-d(x_j, x'_r)) - \mu(-d(x'_m, x'_r)) > v(w, x_m) - v(w, x_j)$$

and since $d(x_j, x'_r) > d(x_j, x_r)$

$$\mu(-d(x_j, x_r)) - \mu(-d(x'_m, x'_r)) > v(w, x_m) - v(w, x_j)$$

then as long as $d(x_m, x_r) > d(x'_m, x'_r)$, w will also satisfy

$$\mu(-d(x_j, x_r)) - \mu(-d(x_m, x_r)) > v(w, x_m) - v(w, x_j)$$

that is $w \in A_j$, and thus $A_j \subset A_{j'}$ and so

$$\sigma'_j = \int_{A'_j} P_{\mathbf{W}}(d\mathbf{w}) > \int_{A_j} P_{\mathbf{W}}(d\mathbf{w}) = \sigma_j$$

Proposition 2

PROOF:

Consider the case where there exists n reference products.

Let $C_r \subset [0, 1]$ be the set of consumers that have reference product r

The market share σ_j , is defined as follows

$$\sigma_j = \sum_{r=1}^n \left(\int_{A_{jn}} P_{\mathbf{W}}(d\mathbf{w}) \right) M(C_r)$$

Where $M(\cdot)$ is the Lebesgue measure and $A_{jn} = \{w \in \mathbf{W} : U(w, x_j; x_r) > U(w, x_n; x_r) \forall n \in J\}$.

Let r^* indicate the set such that for all $r \in \{1, \dots, n\}$ $M(C_{r^*}) > M(C_r)$

Let $\sigma_{j_{r^*}} = \left(\int_{A_{j_{r^*}}} P_{\mathbf{W}}(d\mathbf{w}) \right)$

As shown in proposition 1, we can increase x_{r^*} to x'_{r^*} with $d(x_j, x'_{r^*}) < d(x_j, x_{r^*})$ such that $\sigma'_{j_{r^*}} > \sigma_{j_{r^*}}$

Then by assumption 5 $\sigma'_{j_r} \leq \sigma_{j_r}$ for all $r \in \{1, \dots, n\} \setminus \{r^*\}$

As long as $M(C_r)(\sigma'_{j_r} - \sigma_{j_r}) > \sum M(C_n) (\sigma'_{j_n} - \sigma_{j_n})$,

$$\sigma'_j > \sigma_j$$

Note that since $M(C_r)$ is decreasing in n . This results hold for a sufficiently small n .

However as $n \rightarrow \infty$, $M(C_r) \rightarrow 0$. and thus $\sigma'_j \leq \sigma_j$

Hence if $\sigma'_j > \sigma_j$ for a small n and $\sigma'_j \leq \sigma_j$ for a large n , then there must exist a threshold \bar{R}