Econ 455 Discussion Section-Handout 10

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04/21/2016

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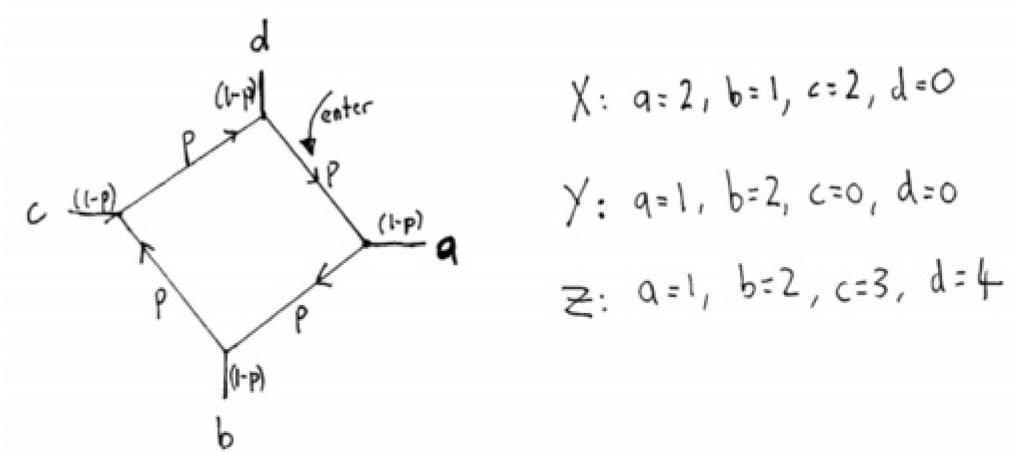
1 Bounded Rationality:

1. You're driving clockwise around a roundabout (traffic circle). You're a bit forgetful in that you can't tell the four exits on the roundabout from eachother. You know where you got on, but you can't see it each time you pass it, so you can't use that as a point of reference, and you don't remember how many exits you've already passed. Since you can't tell the exits apart, all you can do is choose p where, at each exit, p is the probability you stay on the roundabout and (1 − p) is the probability you get off. All of this is shown in the picture below (albeit with a rather square roundabout). You're planning (in an ex ante sense) what p would be best for this situation.



Note that X, Y, Z describe three different iterations of this situation, each with different payoffs

(a) Use intuition to find the optimal p for roundabout X.
He can get the maximal possible payoff by always exiting immediately, i.e. p = 0, so this is optimal.
Note that $V(p) = (1-p)2 + p(1-p)1 + p^2(1-p)2 + p^3(1-p)(0) + p^4V(p)$, so $V(p) = \frac{2-p-+p^2-2p^3}{1-p^4}$
Hence this gets maximized if $p^* = 0$

(b) Use intuition to guess an approximately optimal p for roundabout Y
Here if p = 0, then you're always getting 1, but you might be tempted to pick p > 0 so that you can sometimes get 2. We can actually argue that p = 0 is not ex ante optimal by noting that it isn't interim optimal, since the two have to coincide. If you were always playing p = 0, then given that you're at a turnoff, you know you're at the first turnoff (the one on the right). Then, by deviating to p = 1, assuming you'll stick with p = 0 in the future (the assumption we make when checking interim optimality, you could get a payoff of 2 for certain, and therefore there is an incentive to deviate and p = 1 is not interim optimal. If we want to actually solve it, we can compute it again.
Note that $V(p) = (1-p)1 + p(1-p)2 + p^2(1-p)0 + p^3(1-p)0 + p^4V(p)$.
Hence $V(p) = \frac{1-p-2p^2}{1-p^4}$, which is maximized at $p \approx .27$

(c) Use intuition to argue that there is no optimal p for roundabout Z. But also argue that we know roughly what that optimal p should be.

The intuition here is a little more complicated. First, note that no matter what $p \in [0, 1)$ we choose, we are more likely to to end up turning off

at the intersections that come sooner than we are to get off at those that come later. So we're more likely to get 1 than we are 2, 2 more likely than 3 and 3 more likely than 4. The reasoning is simple: we get 1 with probability (1 – p). We get 2 with probability p(1 – p) and we know $p(1-p) < (1-p)$ for all$p \in (0, 1)$. But note that as p gets closer to 1, the probability we get 2 actually approaches the probability we get 1. In fact, as p goes closer and closer to 1, we essentially expect to go round and round and round, so the degree to which 1 is more likely than 2 decreases and decreases. In the limit, as p goes to 1, all four payoffs become equally likely, which is essentially what we would like, as the later (and therefore less likely for$p \in [0, 1)$) payoffs are better. So the higher the p the better. However, if p = 1, are payoffs are actually undefined, because we never actually turn off. So, roughly speaking, the optimal p is a p that's as close to 1 as possible without equalling 1. Of course, that exact p does not exist. For any $p < 1$. $\exists p'$s.t. $p < p' < 1$, and that's the technical reason why no solution exists, even though we know roughly what it is.

If we want to do this with math $V(p) = (1-p)1 + p(1-p)2 + p^2(1-p)3 + p^4(1-p)4 + p^4V(p)$., solve for $V(p)$ and maximize it subjet to $0 \leq p \leq 1$, then you will find $p^* = 1$, which does not identifies the payoff

(d) Set up the equation that you would maximize to solve for the optimal p.

We just did for each previous part above

2. You are able to go to two stores to purchase a particular television you want to buy. Your prior belief is that, at each store, the TV could be priced anywhere between 100 and 300 dollars, and any dollar amount in that range is equally likely. Also, you believe that the pricing between the stores is independent, so even if the price at one store is 100, it is still just as likely that the price at the other store will be 300 (or any other $p \in (100, 300]$) as it will be 100. Unfortunately, you're a bit spacey and can't remember the exact price at the first store when you visit the second – you can only remember whether you perceived the price as low or high, and have to decide whether to go back and purchase at the first store or purchase at the second given that memory. You must purchase it from one of the two stores. Assume you're risk-neutral.

(a) Suppose you define $p_1 \in [100, 250)$ as low and $p_1 \in [250, 300]$ as high. Further suppose that you're now at the second store and you remember that the price at the first store was high. For what p_2 will you choose to purchase from the second store?

You'll purchase at the second store if the price is lower than the expected price at the first score. That is, you'll purchase at store 2 if:$p_2 \leq E[p_1|p_1\, is\, high]$ or

$p_2 \leq E[p_1|p_1 \in [250, 300]]$that is$p_2 \leq 275$

(b) What if you instead remember that the price at the first store was low? For what p_2 will you choose to purchase from the second store?

Same again That is, you'll purchase at store 2 if:$p_2 \leq E[p_1|p_1\, is\, low]$ or

$p_2 \leq E[p_1|p_1 \in [100, 250)]$that is$p_2 \leq 175$

(c) What is your ex ante expected payment given your partitioning of the price space (i.e$p_1 \in [100, 250)$ as low and $p_1 \in [250, 300]$ as high)?

The probability that you perceive the first price as low is $Pr(p_1 < 250) = 3/4$. If you perceive the first price as high, then you would purchase at store two is $p_2 \leq 275$ which occurs with probability $Pr(p_2 \leq 275) = 7/8$ In this case, the expected payment is $E[p_2|p_2 \in [100, 275]] = \frac{375}{2}$. and with prob $1/8$, $p_2 \geq 275$ and you go back to store 1 and pay an expected price of 275. That gives us the first part of the expected payment (the part coming from perceiving the first price as high) That gives us the first part of the expected payment (the part coming from perceiving the first price as high)

$Expected\, Payment = \frac{1}{4}(\frac{7}{8} \times \frac{375}{2} + \frac{1}{8} \times 275) + ...$ We now need to add on what happens when we perceive p_1 as low, which happens with prob 3/4 We would choose to purchase at store 2 if $p_2 \leq 175$ which happens with probability3/8, in which case the expected payment is $\frac{100+175}{2} = \frac{275}{2}$ With complementary probability 5/8, we go back and purchase at store one where the expected payment is 175. Adding that in gives us the following: $Exected\, payment = \frac{1}{4}(\frac{7}{8} \times \frac{375}{2} + \frac{1}{8} \times 275) + \frac{3}{4}(\frac{3}{8} \times \frac{275}{2} + \frac{5}{8} \times 175) = \frac{2725}{16}$

(d) Recall from the lectures that to find the optimal 2-category memory, we solve $\lambda^* = E[p_2|p_2 \in (E_L, E_H)]$where λ^* was the optimal cutoff price between low and high, andE_L and E_H were the average prices amongst those classified as low and high respectively. Solve for the optimal 2-category memory (i.e. a partitioning of [100, 300] into two intervals).

if λ^* is the cutoff, then the low interval is $[100, \lambda^*]$ and the high interval is $[\lambda^*, 300]$. Therefore $E_L = \frac{100+\lambda^*}{2}$ $E_H = \frac{\lambda^*+300}{2}$, pluggin this gives us $\lambda^* = E\left[p_2 | p_2 \in (\frac{100+\lambda^*}{2}, \frac{\lambda^*+300}{2})\right]$, Now, given that p2 is drawn from a uniform distribution on [100, 300], but we're conditioning on it being between $E_L = \frac{100+\lambda^*}{2}$ $E_H = \frac{\lambda^*+300}{2}$, now think anywhere between those bounds is equally likely, so the expectation of p_2 given that condition $\lambda^* = \frac{\frac{100+\lambda^*}{2} + \frac{\lambda^*+300}{2}}{2}$, if we solve for $\lambda*$, whe get $\lambda^* = 200$, So the optimal 2-category memory is low$[100, 200]$ and high (200, 300].

(e) What is your ex ante expected payment given the optimal 2-category memory you found in (d)? Compare it to that in (c).

Using identical analysis to that in part (c) (except with the cutoff at 200) gives us: $Exected\,payment = \frac{1}{2}(\frac{3}{4} \times \frac{350}{2} + \frac{1}{4} \times 250) + \frac{1}{2}(\frac{1}{4} \times \frac{250}{2} + \frac{3}{4} \times 150) = \frac{675}{4}$, and, as you can see, we ex ante expect to pay slightly less when we have our 2-category memory set optimally.