Econ 455 Discussion Section-Handout 11

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1. You are able to go to two stores to purchase a particular television you want to buy. Your prior belief is that, at each store, the TV could be priced anywhere between 100 and 300 dollars, and any dollar amount in that range is equally likely. Also, you believe that the pricing between the stores is independent, so even if the price at one store is 100, it is still just as likely that the price at the other store will be 300 (or any other $p \in (100, 300]$) as it will be 100. Unfortunately, you're a bit spacey and can't remember the exact price at the first store when you visit the second – you can only remember whether you perceived the price as low or high, and have to decide whether to go back and purchase at the first store or purchase at the second given that memory. You must purchase it from one of the two stores. Assume you're risk-neutral.

   (a) Suppose you define $p_1 \in [100, 250)$ as low and $p_1 \in [250, 300]$ as high. Further suppose that you're now at the second store and you remember that the price at the first store was high. For what p_2 will you choose to purchase from the second store?

   (b) What if you instead remember that the price at the first store was low? For what p_2 will you choose to purchase from the second store?

   (c) What is your ex ante expected payment given your partitioning of the price space (i.e$p_1 \in [100, 250)$ as low and $p_1 \in [250, 300]$ as high)?

   The probability that you perceive the first price as low is $Pr(p_1 < 250) = 3/4$. If you perceive the first price as high, then you would purchase at store two is $p_2 \leq 275$ which occurs with probability $Pr(p_2 \leq 275) = 7/8$ In this case, the expected payment is $E[p_2|p_2 \in [100, 275]] = \frac{375}{2}$. and with prob $1/8$, $p_2 \geq 275$ and you go back to store 1 and pay an expected price of 275. That gives us the first part of the expected payment (the part coming from perceiving the first price as high) That gives us the first part of the expected payment (the part coming from perceiving the first price as high)
   $Expected\,Payment = \frac{1}{4}(\frac{7}{8} \times \frac{375}{2} + \frac{1}{8} \times 275) + ...$ We now need to add on what happens when we perceive p_1 as low, which happens with prob 3/4 We would choose to purchase at store 2 if $p_2 \leq 175$ which happens with probability3/8, in which case the expected payment is $\frac{100+175}{2} = \frac{275}{2}$ With complementary probability 5/8, we go back and purchase at store one where the expected payment is 175. Adding that in gives us the following: $Exected\,payment = \frac{1}{4}(\frac{7}{8} \times \frac{375}{2} + \frac{1}{8} \times 275) + \frac{3}{4}(\frac{3}{8} \times \frac{275}{2} + \frac{5}{8} \times 175) = \frac{2725}{16}$

   (d) Recall from the lectures that to find the optimal 2-category memory, we solve $\lambda^* = E\left[p_2|p_2 \in (E_L, E_H)\right]$where λ^* was the optimal cutoff price between low and high, andE_L and E_H were the average prices amongst those classified as low and high respectively. Solve for the optimal 2-category memory (i.e. a partitioning of [100, 300] into two intervals).

   if λ^* is the cutoff, then the low interval is $[100, \lambda^*]$ and the high interval is $[\lambda^*, 300]$. Therefore $E_L = \frac{100+\lambda^*}{2}$ $E_H = \frac{\lambda^*+300}{2}$, pluggin this gives us $\lambda^* = E\left[p_2|p_2 \in (\frac{100+\lambda^*}{2}, \frac{\lambda^*+300}{2})\right]$, Now, given that p2 is drawn from a uniform distribution on [100, 300], but we're conditioning on it being between $E_L = \frac{100+\lambda^*}{2}$ $E_H = \frac{\lambda^*+300}{2}$, now think anywhere between those bounds is equally likely, so the expectation of p_2 given that condition $\lambda^* = \frac{\frac{100+\lambda^*}{2} + \frac{\lambda^*+300}{2}}{2}$, if we solve for $\lambda*$, whe get $\lambda^* = 200$, So the optimal 2-category memory is low[100, 200] and high (200, 300].

   (e) What is your ex ante expected payment given the optimal 2-category memory you found in (d)? Compare it to that in (c).

   Using identical analysis to that in part (c) (except with the cutoff at 200) gives us: $Exected\,payment = \frac{1}{2}(\frac{3}{4} \times \frac{350}{2} + \frac{1}{4} \times 250) + \frac{1}{2}(\frac{1}{4} \times \frac{250}{2} + \frac{3}{4} \times 150) = \frac{675}{4}$, and, as you can see, we ex ante expect to pay slightly less when we have our 2-category memory set optimally.

2. Source Amnesia: Suppose that you want to find out the true intentions of Donald Trump. He could have good intentions G, or bad intentions B. If you recieve a good signal you think that there is a p chance that he has good intentions. In the same way if you read something bad about him, then there is a p chance that he has bad intentions.

   (a) Suppose that yesterday you read on the Badger Herald that Donald Trump is a true gentleman. Today you discovered at his twitter account that he really cares about the environment. What is your posterior likelihood of your beliefs? $\frac{P(G|gg)}{P(B|gg)} = \frac{P(gg|G)}{P(gg|B)} = \frac{p^2}{(1-p)^2}$

   (b) Actually, the Badger Herald used the same tweet you saw to argue that he is a true gentleman. You realize that, hence what is the likelihood of your beliefs? $\frac{P(G|g)}{P(B|g)} = \frac{P(g|G)}{P(g|B)} = \frac{p}{(1-p)}$

   (c) Now suppose that you do not know whether the tweet you saw was new or old information. However you know that there is a λ chance that what you read was new information, and $1-\lambda$ that it was old. Write down your new beliefs. $\frac{p^2\lambda+(1-\lambda)p}{(1-p)^2\lambda+(1-\lambda)p} = \frac{p}{(1-p)} \times \frac{\lambda p+(1-\lambda)}{\lambda(1-p)+(1-\lambda)}$

   (d) Suppose that it was new information, what implications have source amnesia?

   You underestimate the facts that he might have good intentions

   (e) Suppose it is old information, what implications have source amnesia?

   You overestimate the fact, so he might not have good intention

3. Suppose people lump all vehicles (in particular, airplanes (a), boats (b), and cars (c)) into one product category. Further suppose there are two types of consumers, which we'll refer to as the "unwashed masses (M)" and the "one-percenters (O)" respectively. While both groups lump together the three types of vehicles, the three vehicles have different salience levels to each group as follows:

$$p^M = (p_a^M, p_b^M, p_c^M) = \left(0, \frac{1}{4}, \frac{3}{4}\right) \quad p^O = (p_a^O, p_b^O, p_c^O) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$$

Let q_a, q_b and q_c denote the quality of an airplane, boat and car respectively, and consider that they are a function of wing length x, pontoon circumference y, and radio antenna length z respectively in the following way: $q_a = 3x$, $q_b = 2y$, $q_c = z$. Assume that each vehicle will sell for its perceived quality, and that the costs of building wings, pontoons and radio antennas are: $c(x) = x^2$, $c(y) = 2y^2$, $c(z) = 3z^2$.

   (a) For just this part, assume that agents actually don't group the three vehicles. Find the optimal x, y, z (to maximize profits) for each of a, b, and c.

   Note immediately that we're not going to be putting wings and pontoons on cars because they are costly and don't affect the quality (and therefore the price) of the cars. For the airplane, we want to build wings up until the point where the marginal cost of extending the wing equals the marginal benefit.That is, we set $3 = 2x \implies x^* = \frac{2}{3}$, $2 = 4y \implies y^* = 1/2$ and $1 = 6z \implies z^* = 1/6$

   (b) Going forward, assume the agents do group the three vehicles into one category. Explain why we might want to put pontoons and/or wings onto cars?

   If people group the three modes of transport together, then they associate bigger pontoons and longer wings with better boats and planes, and since they group the three, they transfer those properties onto cars also. By putting pontoons and wings on cars, we may increase the value of those cars due to this transference, even though the cars are actually not made better off by pontoons and wings.

   (c) You are asked to design the optimal car (in that it maximizes profits) to sell to the masses. What is it?

   Sinces the masses put salience $p_b^M = \frac{1}{4}$ and $p_c^M = \frac{3}{4}$, we want to maximize. $\pi = 0(3x) + \frac{1}{4}(2y) + \frac{3}{4}(z) - x^2 - 2y^2 - 3z^2$

   Then $F.O.C._y$ $\frac{1}{2} - 4y = 0$, $y = \frac{1}{8}$, $F.O.C._z$ $\frac{3}{4} - 6z = 0$, $z^* = \frac{1}{8}$, $x* = 0$

   (d) Find the optimal (in that it maximizes profits) boat to sell to the masses (hint: you do not need to do any math).

   It's just the same as the optimal car. You can try to set it up and you'll see you're writing the same equation, since the buyers group the two products in any case

   (e) Find the optimal airplane/boat/car to sell to the one-percenters (O).

   Similar to c, $x = \frac{3}{4}$, $y = \frac{1}{8}$, $z = \frac{1}{24}$

(f) Finally, suppose half of the population are type O (as illogical as that may be) and half are type M and you can only design one boat that you must sell to both types. Find the optimal (i.e. profit-maximizing) boat.

just take the average between e and f to get $x = \frac{3}{8}$, $y = \frac{1}{8}$, $z = \frac{1}{12}$

(g) Suppose that agents can actually tell the difference between boats and airplanes, but aren't sure whether a car is really closer to an airplane or a boat. A recent study came out that said that air travel was significantly safer and a lot more fun than boating, even more so than thought previously. If you were building cars, describe (qualitatively) how this might affect your choice of x, y, z? Use a framing argument.

Slap some wings on that car and call it a 747 to frame it as an airplane rather than a boat. Reduce the pontoon size to zero, to make sure it's not viewed as a boat (and save costs). It won't really affect the size of the radio antenna you choose.