Econ 441 Discussion Section-Handout 3

Emilio Cuilty

09/25/2015

1 Office Hours

Office Hours	Tuesday	11:00 P.M.-1:00 P.M.	Social Sciences 6470

2 Problems:

1. **Free Riding on a Homework:** Suppose that in your next econ 441 you are asked to work in pairs, you only have to submit one assigment. In total the homework is given by $H = H_y + H_o$, where H_y represents your contribution, and $H_o$represents the other guy contribution. Consider that the day has 16 possible work hours, and that you can spend them in leisure L_y or working on the homework, H_y. Your utility is given by the following utility function$U_y = \alpha ln(L_y) + (1-\alpha)ln(H)$, and you know the other guy has a utility function given by $U_o = L_o^{\alpha} H^{1-\alpha}$

   (a) Do you have different preferences than the other guy?
   No, both are Cobb-Douglas preferencess

   (b) If you decide to work on your own assuming that your pair will work too, find your best response.
   First note that your problem is to $Max\ U_y = \alpha ln(L_y) + (1-\alpha)ln(H)\ S.T.\ H = H_y + H_o\ 16 = H_y + L_y$.
   Therefore it can be rewritten as $Max\ U_y = \alpha ln(L_y) + (1-\alpha)ln(H_y + H_o)\ S.T.\ 16 = H_y + L_y$
   now your marginal rate of substitution is $MRS = \frac{(1-\alpha)(L_y)}{\alpha(H_y+H_o)}$, and the price relationship is one.
   Therefore $MRS = 1$, or $\frac{(1-\alpha)(L_y)}{\alpha(H_y+H_o)} = 1$, solving for $L_y = \frac{\alpha(H_y+H_o)}{1-\alpha}$.
   Using the budget line, $\frac{\alpha(H_y+H_o)}{1-\alpha} + H_y = 16$, So the best response $H_y = 16(1-\alpha) - \alpha H_o$

   (c) Find the other guys best response, and obtain the Nash Equilibrium
   Since the problem is symmetric $H_o = 16(1-\alpha) - \alpha H_y$, Thus the Nash Equilibrium $H_y = 16(1-\alpha) - 16\alpha(1-\alpha) + \alpha^2 H_y$,
   Thus $H_y = \frac{16(1-\alpha)}{1+\alpha}$, By symmetry $H_o = \frac{16(1-\alpha)}{1+\alpha}$

   (d) If you agree to work together, what is the efficient amount of hours worked?
   Now you need to add the Marginal rate of substituions, $MRS_y = \frac{(1-\alpha)(L_y)}{\alpha(H)}$, and $MRS_o = \frac{(1-\alpha)(L_0)}{\alpha(H)}$, $SMRS = \frac{(1-\alpha)(L_0+L_y)}{(\alpha)(H)}$,
   Now $SMRS = 1$, $\frac{(1-\alpha)(L_0+L_y)}{\alpha(H)} = 1$, so $\frac{\alpha H}{1-\alpha} - L_Y = L_o$, now you want to use the budget line,
   Add the budget lines so $H + L_Y + L_o = 32$, so $H = 32(1-\alpha)$

2. **Charity as a Public Good (Revisited):** Simon and Garfunkel will give a charity concert, however not all the money will go to charity. Simon truly cares about poor people his utility over own money m and money spent on charity (c) is $U_S = m_S + 2(C)^{1/2}$, Garfunkel cares a lot less and his utility function is $U_G = m_G + C^{1/2}$. After giving it to charity, each will obtain some amount of money w for the concert

(a) If they consider Charity as a public good, but they want to act individually, find the best response functions

$Max\ U_S = m_S + 2(C)^{1/2}\ s.t.\ C_S + C_G = C\ C_S + m_S = w$, and $Max\ U_G = m_G + 2(C)^{1/2}\ s.t.\ C_S + C_G = C\ C_G + m_G = w$,

Rewrite $Max\ U_S = m_S + 2(C_S + C_G)^{1/2}\ C_S + m_S = w$, and $Max\ U_G = m_G + 2(C_S + C_G)^{1/2}\ s.t.\ C_G + m_G = w$,

Then the $MRS_S = \frac{2}{(C_S+C_G)^{1/2}}$ and $MRS_G = \frac{1}{(C_S+C_G)^{1/2}}$ the price relationship is the same for both.

So $\frac{2}{(C_S+C_G)^{1/2}} = 1$ and $\frac{1}{(C_S+C_G)^{1/2}} = 1$ then the best responses are $C_S = 4 - C_G$ and $C_G = 1 - C_S$

(b) Is there a Pure Nash Equilibrium?

No, Since the system of equations of best responses does not have a solution

(c) What is the efficient level of C. Is it unique?

The first thing that you have to do is find the marginal rates of substitution.

$MRS_S = \frac{c^{-1/2}}{1}$ and $MRS_G = \frac{c^{-1/2}}{2}$. Then you use the Samuelson Condition (Adding the MRS), $\sum MRS = \frac{c^{-1/2}}{1} + \frac{c^{-1/2}}{2} = Price\ Relation = 1$

Thus $c^* = \frac{9}{4}$. It is unique

(d) Suppose a that their manager force them to provide the efficient level of c. What shares (S_s, S_G) of the total cost of this provision can the manger assign to Simon and Garfunkel so that each find it privately optimal to pay for this share?

We know that $c^* = \frac{9}{4}$, thus by the individual MRS of each guy, $MRS_S(c^*) = \frac{2}{3} = S_s$ and $MRS_G(c^*) = \frac{1}{3} = S_G$

(e) Now suppose that $U^s = m_s c^2$ and$U^G = m_G c$, Describe the efficient level of c. Is it unique?

Now, you know that $m^S + m^G + c = 2w$, so $c = 2m^S + (2w - m^s - c)$, so $c^* = \frac{m^s}{2} + w$, Since it depends on who provides the good there are multiple solutions.

(f) With this preferences, find the best responses if they decide to act separately.

Again consider $Max\, U^s = m_s(c_S + c_G)^2\ s.t.\ m_S + c_S = 1$ and $Max\, U^G = m_G(c_S + c_G)\ s.t.\ m_S + c_S = 1$,

then $MRS_S = \frac{2m_S}{(C_S+C_G)}$ and $MRS_G = \frac{m_G}{(C_S+C_G)}$.

Therefore solving for $m_s\ or\ m_g$ $m_s = \frac{C_S+C_G}{2}$ and $m_G = c_S + c_G$, using their budget line

$c_s + \frac{c_s+c_g}{2} = w$ and $c_s + c_g + c_g = w$. Thus the best responses are

$c_s = \frac{2w-c_g}{3}$and $c_g = \frac{w-c_S}{2}$

(g) Is there a Pure Nash Equilibrium?

Yes and it can be found by using the best responses, $3c_s = 2w - \frac{w-c_S}{2}\ c_s = \frac{3w}{8}$ and $c_g = \frac{4w}{16}$

3. Vermillion City is considering building a public subway system to help relieve urban traffic congestion. Vermillion politicians in favor of the project argue that the system would benetffit the population by drastically reducing commuting time, not to mention the jobs the project would create during the subway construction process.There is also Celadon City (a neighboring city to Vermillion City) which built a subway system in 2001 similar to the one proposed in Vermillion City. The following chart shows total annual commuting hours for both cities in the year 2000 and 2002:

	2000	2002
Vermillion City	40,000	45,000
Celadon City	30,000	25,000

(a) Using the available data, construct the best estimate of the effect of the subway on commuting hours in Celadon City. Describe any concerns about the validty of this estimate.

Estimated effect of subway on commuting hours is: (25,000-30,000) - (45,000-40,000) = - 10,000.The claim we make is that these two towns are similar and that Vermillion City represents what would have happened in Celadon if Celadon didn't implement the subway system. We will use this in our benet analysis in the next part.

(b) Labor and construction materials are needed to build the subway. Vermillion's labor market is competitive, and the wage is $20/hour. Total labor costs would come to $1,500,000. However, monopolies own the materials factories, and therefore material costs would come to $2,500,000. A reputable economist has estimated that the cost of materials would fall to $1,800,000 if they could be purchased in a competitive market. All costs would be paid immediately and the subway would be built instantaneously. After project completion, Vermillion City would annually experience the time-savings reduction in commuting hours found in part 1. (If you didn't nd this, plug it in as X). If the interest rate is r= 5%, calculate the PDV of this project and use this value to recommend whether or not the subway system should be built.

Labor costs are $1.5 million. The materials costs are $2.5 million but because the opportunity cost of using the same materials in a competitive market framework would be $1.8 million, the economic costs are then $1.8 million. So total costs are $3.3 million

The benetfit comes from the effect of the subway system on commuting hours. We quantify it into dollars by multiplying it by the per hour wage $20. So the annual benet is ($20)(10,000) = $200,000. But we want to nd the PDV of all future benets this has which will extend forever. This means that the total benets are going to be $\sum_{t=1}^{\infty} \frac{200{,}000}{1.05^t} = \frac{200{,}000}{.05} = 4million$. So the benefits outweigh the costs, meaning this is a good investment.