Econ 455 Discussion Section-Handout 4

Emilio Cuilty

02/19/2016

---

1 Confirmation Bias and Expected Utility:

1. **Confirmation Bias:** A professor is trying to decide if a student is good or bad. The prior belief that the student is good is 1/2. Then the professor receives signals (tests), where g represents a good test performance and b represents a bad test performance. Once a professor believes it more than 50% likely that a student is good (bad), she will misinterpret a bad (good) test performance as good (bad) with probability q. Also good students do good on tests and bad students do bad on tests with$P(g|G) = P(b|B) = p$.

   (a) Suppose $q = 1/2$ and $p = 3/4$. Given a naive professor observe $\tilde{g}\tilde{g}\tilde{g}$, what is probability that the student is good?
   $\frac{P(G|\tilde{g}\tilde{g}\tilde{g})}{P(B|\tilde{g}\tilde{g}\tilde{g})} = \frac{P(\tilde{g}\tilde{g}\tilde{g}|G)}{P(|\tilde{g}\tilde{g}\tilde{g}|B)}\frac{P(G)}{P(B)}$, now $P(G) = P(B) = \frac{1}{2}$.
   $P(\tilde{g}\tilde{g}\tilde{g}|G) = p^3 = (\frac{3}{4})^3 = \frac{27}{64}$ $P(\tilde{g}\tilde{g}\tilde{g}|B) = (1-p)^3 = (\frac{1}{4})^3 = \frac{1}{64}$.
   So $\frac{P(G|\tilde{g}\tilde{g}\tilde{g})}{P(B|\tilde{g}\tilde{g}\tilde{g})} = 27$, now $P(G|\tilde{g}\tilde{g}\tilde{g}) + P(B|\tilde{g}\tilde{g}\tilde{g}) = 1$, solving the system of equations you get that $P(G|\tilde{g}\tilde{g}\tilde{g}) = \frac{27}{28}$

   (b) Now do the same exercise for a sophisticated professor.
   $\frac{P(G|\tilde{g}\tilde{g}\tilde{g})}{P(B|\tilde{g}\tilde{g}\tilde{g})} = \frac{P(\tilde{g}\tilde{g}\tilde{g}|G)}{P(|\tilde{g}\tilde{g}\tilde{g}|B)}\frac{P(G)}{P(B)}$,
   Step by step
   $P(\tilde{g}\tilde{g}\tilde{g}|G)$,
   Given your prior was 1/2 and you perceived the first test as good, it must have been a good test performance. However, either of the second and third tests could have been bad and misread.
   Therefore $P(\tilde{g}\tilde{g}\tilde{g}|G) = P(ggg|G) + P(gbg|G)P(\tilde{g}|b) + P(bgg|G)P(\tilde{g}|b) + P(bbg|G)P(\tilde{g}|b)^2$
   or $P(\tilde{g}\tilde{g}\tilde{g}|G) = \frac{27}{64} + 2(\frac{9}{64})(\frac{1}{2}) + \frac{1}{64}(\frac{1}{2})^2 = \frac{145}{256}$
   Now $P(\tilde{g}\tilde{g}\tilde{g}|B)$,
   Given your prior was 1/2 and you perceived the rst test as good, it must have been a good test performance. However, either of the second and third tests could have been bad and misread.
   Therefore $P(\tilde{g}\tilde{g}\tilde{g}|B), = P(ggg|B) + P(gbg|B)P(\tilde{g}|b) + P(bgg|B)P(\tilde{g}|b) + P(bbg|B)P(\tilde{g}|b)^2$
   or $P(\tilde{g}\tilde{g}\tilde{g}|B), = \frac{1}{64} + 2(\frac{3}{64})(\frac{1}{2}) + \frac{9}{64}(\frac{1}{2})^2 = \frac{25}{256}$.
   So$\frac{P(G|\tilde{g}\tilde{g}\tilde{g})}{P(B|\tilde{g}\tilde{g}\tilde{g})} = \frac{\frac{145}{256}(\frac{1}{2})}{\frac{25}{256}(\frac{1}{2})} = \frac{29}{5}$, using the aditional equation that $P(G|\tilde{g}\tilde{g}\tilde{g}) + P(B|\tilde{g}\tilde{g}\tilde{g}) = 1$ we get that $P(G|\tilde{g}\tilde{g}\tilde{g}) = \frac{29}{34}$.
   While that's still pretty high, it's signicantly lower than the Naive case

   (c) Suppose the sophisticated professor's prior probability that the student is good is 50/100 What is her posterior given she observes $\tilde{g}\tilde{g}$?
   Since the professor started o unbiased, the frst test results must have been a g. The second could have been a b and been misinterpreted.
   Therefore $P(\tilde{g}\tilde{g}|G) = \frac{9}{16} + \frac{3}{16}\frac{1}{2} = \frac{21}{32}$ and $P(\tilde{g}\tilde{g}|B) = \frac{1}{16} + \frac{3}{16}\frac{1}{2} = \frac{5}{32}$
   So $\frac{P(G|\tilde{g}\tilde{g})}{P(B|\tilde{g}\tilde{g})} = \frac{\frac{21}{32}(\frac{1}{2})}{\frac{5}{32}(\frac{1}{2})} = \frac{21}{26}$

(d) Suppose the sophisticated professor's prior probability that the student is good is $\frac{51}{100}$What is her posterior given she observes $\tilde{g}\tilde{g}$?

Now the professor is biased even before seeing the first test result.

Therefore, it could be that the student actually did poorly on either or both of the tests and it was misperceived,

$P(\tilde{g}\tilde{g}|G) = \frac{9}{16} + 2\frac{3}{16}\frac{1}{2} + \frac{1}{16}(\frac{1}{2})^2 = \frac{49}{64}$ $P(\tilde{g}\tilde{g}|B) = \frac{1}{16} + 2\frac{3}{16}\frac{1}{2} + \frac{9}{16}(\frac{1}{2})^2 = \frac{25}{64}$

and $\frac{P(G|\tilde{g}\tilde{g})}{P(B|\tilde{g}\tilde{g})} = \frac{\frac{49}{64}(\frac{51}{100})}{\frac{25}{64}(\frac{49}{100})} = \frac{51}{25}$ and it follows that $P(G|\tilde{g}\tilde{g}) = \frac{51}{76}$

(e) Compare your answers to (c) and (d). What is counter-intuitive here? Where is it coming from in the model?

It's counter-intuitive that the posterior in (d) is lower than the posterior in (c). The professor observed the same signals in both cases and had a higher prior in (d), yet her posterior in (d) is lower than in (c). What's generating this in the model is that when the sophisticated professor has a prior of 1/2, she trust that the rst signal she receives is an accurate re ection of the test performance. In (d) however, even though she initially believes the student is more likely to be good, that belief (and the bias it generates) causes her to doubt the accuracy of the rst signal she receives.

(f) What is the professor's posterior if it is equally likely that good students and bad students do well on tests and she has no bias? Answer in words, appealing to the likelihood-ratio formulation of Bayes' Rule above.

Then P(n|A) = P(n|B), so the posterior just equals the prior, i.e. the evidence is uninformative.

2. **Review of Expected Utility:**

(a) Final Outcomes only matter, Consider that it is possible to win (Love of your life ,Dream Job , 1 Billion of Dollars). Your chances are different depending in which country do you live. Suppose that you are considering two possible jobs, the first one is a corporate finance Job. This job will take you to the Moon or to Mexico or to Argentina with prob 1/3 each. In the Moon you will get for sure the love of your life for sure, however this will not be your dream job, or will give you a million dollars. In Mexico or Argentina you have $\frac{1}{4}$ of getting the love of your life, $\frac{3}{8}$ to get your dream job and $\frac{3}{8}$of getting 1 million dollars. Now you can take a public job, and this job will give you the opportunity to go to New york with $\frac{1}{2}$ probability or to San Francisco with probability $\frac{1}{2}$. In New York you have $\frac{1}{2}$ of probability of getting the love of your life, and $\frac{1}{2}$ of probability of getting a million dollars. If you go to San Francisco you get $\frac{1}{2}$ of probability of finding the love of your life, or you can get your dream job with $\frac{1}{2}$ of probability. As a Rational Player, which one will you choose?

Under Expected utility theory choices are made only over final outcomes (Reduced Lotteries).

Which means that if you take the first job, you will have the following probabilities.

For thel love of your life $(\frac{1}{3})(1) + (\frac{1}{3})(\frac{1}{4}) + \frac{1}{3}(\frac{1}{4}) = \frac{1}{2}$, Your Dream job $\frac{1}{3}(0) + \frac{1}{3}(\frac{3}{8}) + \frac{1}{3}(\frac{3}{8}) = \frac{1}{4}$, A million dollars $\frac{1}{3}(0)\frac{1}{3}(\frac{3}{8}) + \frac{1}{3}(\frac{3}{8}) = \frac{1}{4}$

Now under the second job you will have the following probabilities

For thel love of your life $(\frac{1}{2})(\frac{1}{2}) + (\frac{1}{2})(\frac{1}{2}) = \frac{1}{2}$, Your Dream job $\frac{1}{2}(0) + \frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$, A million dollars $\frac{1}{2}(\frac{1}{2}) + \frac{1}{2}(0) = \frac{1}{4}$

In othr words, both jobs will get you the same probabilities over the possible outcomes, you should be indifferent

(b) Which one is bigger $U(\sum_k^K \alpha_k L_k)$ or $\sum_k^K \alpha_k U(L_k)$?

Expected Utility implies that the utility function is linear, $U(\sum_k^K \alpha_k L_k) = \sum^K \alpha_k U(L_k)$

(c) What does independence axiom says?

If $L_1 \prec L_2$, then $\alpha L_1 + (1-\alpha)L_3 \prec \alpha L_2 + (1-\alpha)L_3$

(d) Draw a picture where you show on the vertical axis the utility and on the horizontal axis the amount of money. Draw a typical risk averse Bernoulli utility function, draw a risk lover's as well

(e) What is the Equivalent of certainty? label it on the graph

3. **A Simple Insurance Problem:** Bucky Badger has \$20,000, yet the future is very uncertain for him. In one scenario, nothing will happen and he will be able to spend all his money on consumption. This will happen with probability p. In another scenario, Evil Buckeyes will attack Madison, but he fights back and saves the city, yet he loses \$10,000. The probability of this event is $1-p$. His utility is $u = log(c)$.

   (a) Without any insurance, what is Bucky's expected utility?

   $$EU = Plog(20,000) + (1-p)(10,000)$$

   (b) UW Madison offers him an insurance for the bad scenario. If the Buckeys attack Madison, they will pay back x at a premium of $0.4x$. How much will Bucky will insure to maximize his expected utility?

   $$EU = plog(20,000 - .4x) + (1-p)log(10,000 + x - .4x)$$

   Taking first order conditions, $\frac{-.4p}{20{,}000-.4x} = \frac{.6(1-p)}{10{,}000+.6x}$, $x = \frac{120{,}000-160{,}000p}{2.4}$

   (c) If the premium is actuarially fairly priced, what is the probability of each event?

   The premium has to be equal the expected payoff of the insurer $(1-p)x = .4x$ hence $p = .6$