Econ 455 Discussion Section-Handout 5

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1 Expected Utility VS Prospect Theory:

1. **Review of Expected Utility:**

   (a) **Final outcomes only matter**: Consider that it is possible to win (Love of your life ,Dream Job , 1 Billion of Dollars). Your chances are different depending in which country do you live. Suppose that you are considering two possible jobs, the first one is a corporate finance Job. This job will take you to the Moon or to Mexico or to Argentina with prob 1/3 each. In the Moon you will get for sure the love of your life for sure, however this will not be your dream job, or will give you a million dollars. In Mexico or Argentina you have $\frac{1}{4}$ of getting the love of your life, $\frac{3}{8}$ to get your dream job and $\frac{3}{8}$of getting 1 million dollars. Now you can take a public job, and this job will give you the opportunity to go to New york with $\frac{1}{2}$ probability or to San Francisco with probability $\frac{1}{2}$. In New York you have $\frac{1}{2}$ of probability of getting the love of your life, and $\frac{1}{2}$ of probability of getting a million dollars. If you go to San Francisco you get $\frac{1}{2}$ of probability of finding the love of your life, or you can get your dream job with $\frac{1}{2}$ of probability. As a Rational Player, which one will you choose?

   Under Expected utility theory choices are made only over final outcomes (Reduced Lotteries).

   Which means that if you take the first job, you will have the following probabilities.

   For thel love of your life $(\frac{1}{3})(1) + (\frac{1}{3})(\frac{1}{4}) + \frac{1}{3}(\frac{1}{4}) = \frac{1}{2}$, Your Dream job $\frac{1}{3}(0) + \frac{1}{3}(\frac{3}{8}) + \frac{1}{3}(\frac{3}{8}) = \frac{1}{4}$, A million dollars $\frac{1}{3}(0)\frac{1}{3}(\frac{3}{8}) + \frac{1}{3}(\frac{3}{8}) = \frac{1}{4}$

   Now under the second job you will have the following probabilities

   For thel love of your life $(\frac{1}{2})(\frac{1}{2}) + (\frac{1}{2})(\frac{1}{2}) = \frac{1}{2}$, Your Dream job $\frac{1}{2}(0) + \frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$, A million dollars $\frac{1}{2}(\frac{1}{2}) + \frac{1}{2}(0) = \frac{1}{4}$

   In othr words, both jobs will get you the same probabilities over the possible outcomes, you should be indifferent

   (b) **Linearity:** Which one is bigger $U(\sum_k^K \alpha_k L_k)$ or $\sum_k^K \alpha_k U(L_k)$, where $\sum_{k=1}^K \alpha_k = 1$?

   Expected Utility implies that the utility function is linear, $U(\sum_k^K \alpha_k L_k) = \sum^K \alpha_k U(L_k)$

   (c) **Independence:** What does the independence axiom says?

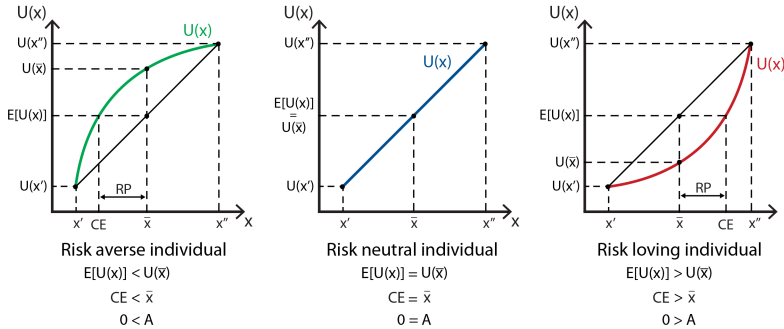
   If $L_1 \prec L_2$, then $\alpha L_1 + (1-\alpha)L_3 \prec \alpha L_2 + (1-\alpha)L_3$

   (d) **Concavity/Convexity:** Draw a picture where you show on the vertical axis the utility and on the horizontal axis the amount of money. Draw a typical risk averse Bernoulli utility function, draw a risk lover's as well

   (e) What is the Equivalent of certainty? label it on the graph

   (f)

Figure 1: d,e



2. **A Simple Insurance Problem:** Bucky Badger has \$20,000, yet the future is very uncertain for him. In one scenario, nothing will happen and he will be able to spend all his money on consumption. This will happen with probability p. In another scenario, Evil Buckeyes will attack Madison, but he fights back and saves the city, yet he loses \$10,000. The probability of this event is $1-p$. His utility is $u = log(c)$.

   (a) Without any insurance, what is Bucky's expected utility?

$$EU = Plog(20,000) + (1-p)(10,000)$$

   (a) UW Madison offers him an insurance for the bad scenario. If the Buckeys attack Madison, they will pay back x at a premium of $0.4x$. How much will Bucky will insure to maximize his expected utility?

$$EU = plog(20,000 - .4x) + (1-p)log(10,000 + x - .4x)$$

Taking first order conditions, $\frac{-.4p}{20,000-.4x} = \frac{.6(1-p)}{10,000+.6x}$, $x = \frac{120,000-160,000p}{2.4}$

   (a) If the premium is actuarially fairly priced, what is the probability of each event?
   The premium has to be equal the expected payoff of the insurer $(1-p)x = .4x$ hence $p = .6$

3. **Loss Aversion:** A person's value function is $v(x) = \sqrt{\frac{x}{2}}$ for gains and $v(x) = -2\sqrt{|x|}$ for losses. The person is facing the choice between a sure \$2 and a 50-50 gamble that pays \$4 if she wins and \$0 if she loses.

   (a) Show algebraically that this person is loss averse, in the sense that she suffers more when she loses \$4 than she benefits when she receives \$4.
   $v(-4) = -4$ while $v(+4) \approx 1.41$. Therefore, losing 4 costs her 4 utils while gaining 4 gives her only 1.41 utils.

   (b) If the outcomes are coded as gains, meaning that she will take the worst possible outcome as her reference point, what is the value of

      i. the sure amount?
      $v(+2) = 1$

      ii. the gamble?

   $v(G) = \frac{1}{2}v(0) + \frac{1}{2}v(+4) = \frac{1}{2}\sqrt{2} \approx: 71$

      i. Which would she prefer?
      The sure amount

   (c) If the outcomes are coded as losses, meaning that she will take the best possible outcome as her reference point, what is the value of

i. . the sure amount?
$v(-2) = -2\sqrt{2} \approx -2.83$

ii. the gamble?
$\frac{1}{2}v(0) + \frac{1}{2}v(-4) = -2$

iii. Which would she prefer?
The Gamble

4. Jimmy wants to put an offer on a house. The house is worth \$200,000 to him. He knows the market pretty well and knows that he is guaranteed to win if he submits an offer of \$150,000 or above. If he bids$b \in [0, 150,000]$, he will win with probability $b/150,000$. If he wins, he pays his bid. Assume he has no outside wealth, so that his assets, x, are worth 200,000–b if he wins and zero if he loses.

(a) Suppose Jimmy is an expected utility maximizer with $u(x) = \sqrt{x}$ risk-loving or risk-neutral? What is his optimal bid?

It is Risk Averse, because the function is concave.

He solves the following problem,

$Max\, P(Wins|b)u(200,000 - b) + P(Loses|b)u(0)$

Then $Max\, \frac{b}{150,000}\sqrt{200,000 - b}$,

Taking First Order Conditions, $\frac{1}{150,000}\sqrt{200,000 - b} - \frac{b}{300,000}(200,000 - b)^{-1/2} = 0$, or $b = 400,000/3$

(b) Suppose Jimmy is an expected utility maximizer with $u(x) = x^2$ risk-loving or risk-neutral? What is his optimal bid? Explain why the answer here is lower than the answer in (a).

He is risk-loving because it is convex.

$max\, \frac{b}{150,000}(200,000 - b)^2$,

Then by the first order conditions $\frac{1}{150,000}(200,000 - b)^2 - \frac{2b}{150,000}(200,000 - b) = 0$, $b = \frac{200,000}{3}$ The answer here is lower because a lower bid in a sense represents a bigger gamble. That is, it lowers the probability you win but increases the payoffs inthe event in which you win. Since Jimmy in (b) is risk-loving, he prefers a larger gamble than that undertaken by Jimmy in (a).

(c) Now suppose that Jimmy is a prospective utility maximizer. His value for gains and losses x is $v(x) = \sqrt{x}$ if $x > 0$ and $v(x) = -\sqrt{-x}$ if $x < 0$. Suppose also that he has a linear probability-weighting function, i.e. $\pi(p) = p$. If his reference point is that he purchases the house for \$150,000, then what is his optimal bid.

If he were to purchase the house for \$150,000, then he would get a utility of \$50,000, so this is the reference point in terms of utility. Now, if hewins the house for his bid b, he will gain 150000–b relative to his reference point.If he loses, however, he will lose 50000 relative to his reference point. Therefore, he solves the following problem.

$Max\, \frac{b}{150,000}(150,000 - b)^{1/2} - (1 - \frac{b}{150,000})^{1/2}(50,000)^{1/2}$

Taking first order conditions we have that $\frac{1}{150,000}(150,000 - b)^{1/2} + \frac{1}{300\sqrt{5}} = \frac{b}{300,000(150,000-b)^{1/2}}$

Solving $b = \frac{100,000}{9}(8 + \sqrt{10}) \approx 124025$

(d) What about if his reference point is that he purchases the house for \$0?

Now, given his reference point is that he buys it for zero, his reference utility is 200,000. Therefore, if he wins with bid b, he will lose b utils relative to his reference point. If he loses, then he will lose 200,000 utils relative to his reference. Therefore he solves the following problem:

$Max\, \frac{-b}{150,000}(b)^{1/2} - (1 - \frac{b}{150,000})^{1/2}(200,000)^{1/2}$, by the F.O.C. $\frac{400\sqrt{5}-3\sqrt{b}}{300,000} = 0$, or $b = \frac{800,000}{9} \approx 88889$

(e) Continue to assume his reference point is that he purchases the house for \$0, but now assume that his probability weighting function is $\pi(p) = p^2$. Solve for his optimal bid

$Max\, (\frac{-b}{150,000})^2(b)^{1/2} - (1 - \frac{b}{150,000})^{1/2}(200,000)^{1/2}$, as before, an with a lot of algebra you get $b \approx 83704$

(f) Now take the approach of Cumulative Prospective Utility and rewrite the maximization problem from (e) in this framework.

$PU = v(x_1) + \pi(P_2)(v(x_2) - v(x_1))$

or $-\sqrt{200,000} + (\frac{-b}{150,000})^2(-\sqrt{b} - (-\sqrt{200,000}))$