Econ 455 Discussion Section-Handout 6

Emilio Cuilty

03/04/2016

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1 Cumulative Prospect Theory:

1. Jimmy wants to put an offer on a house. The house is worth \$200,000 to him. He knows the market pretty well and knows that he is guaranteed to win if he submits an offer of \$150,000 or above. If he bids$b \in [0, 150,000]$, he will win with probability $b/150,000$. If he wins, he pays his bid. Assume he has no outside wealth, so that his assets, x, are worth 200,000–b if he wins and zero if he loses.

   (a) Suppose Jimmy is an expected utility maximizer with $u(x) = \sqrt{x}$ risk-loving or risk-neutral? What is his optimal bid?

   It is Risk Averse, because the function is concave.

   He solves the following problem,

   $Max\, P(Wins|b)u(200,000 - b) + P(Loses|b)u(0)$

   Then $Max\, \frac{b}{150,000}\sqrt{200,000 - b}$,

   Taking First Order Conditions, $\frac{1}{150,000}\sqrt{200,000 - b} - \frac{b}{300,000}(200,000 - b)^{-1/2} = 0$, or $b = 400,000/3$

   (b) Suppose Jimmy is an expected utility maximizer with $u(x) = x^2$ risk-loving or risk-neutral? What is his optimal bid? Explain why the answer here is lower than the answer in (a).

   He is risk-loving because it is convex.

   $max\, \frac{b}{150,000}(200,000 - b)^2$,

   Then by the first order conditions $\frac{1}{150,000}(200,000 - b)^2 - \frac{2b}{150,000}(200,000 - b) = 0$, $b = \frac{200,000}{3}$ The answer here is lower because a lower bid in a sense represents a bigger gamble. That is, it lowers the probability you win but increases the payoffs in the event in which you win. Since Jimmy in (b) is risk-loving, he prefers a larger gamble than that undertaken by Jimmy in (a).

   (c) Now suppose that Jimmy is a prospective utility maximizer. His value for gains and losses x is $v(x) = \sqrt{x}$ if $x > 0$ and $v(x) = -\sqrt{-x}$ if $x < 0$. Suppose also that he has a linear probability-weighting function, i.e. $\pi(p) = p$. If his reference point is that he purchases the house for \$150,000, then what is his optimal bid.

   If he were to purchase the house for \$150,000, then he would get a utility of \$50,000, so this is the reference point in terms of utility. Now, if he wins the house for his bid b, he will gain 150000–b relative to his reference point.If he loses, however, he will lose 50000 relative to his reference point. Therefore, he solves the following problem.

   $Max\, \frac{b}{150,000}(150,000 - b)^{1/2} - (1 - \frac{b}{150,000})^{1/2}(50,000)^{1/2}$

   Taking first order conditions we have that $\frac{1}{150,000}(150,000 - b)^{1/2} + \frac{1}{300\sqrt{5}} = \frac{b}{300,000(150,000-b)^{1/2}}$

   Solving $b = \frac{100,000}{9}(8 + \sqrt{10}) \approx 124025$

   (d) What about if his reference point is that he purchases the house for \$0?

   Now, given his reference point is that he buys it for zero, his reference utility is 200,000. Therefore, if he wins with bid b, he will lose b utils relative to his reference point. If he loses, then he will lose 200,000 utils relative to his reference. Therefore he solves the following problem:

$Max\ \frac{-b}{150,000}(b)^{1/2} - (1 - \frac{b}{150,000})^{1/2}(200,000)^{1/2}$, by the F.O.C. $\frac{400\sqrt{5}-3\sqrt{b}}{300,000} = 0$, or $b = \frac{800,000}{9} \approx 88889$

(e) Continue to assume his reference point is that he purchases the house for \$0, but now assume that his probability weighting function is $\pi(p) = p^2$. Solve for his optimal bid
$Max\ (\frac{-b}{150,000})^2(b)^{1/2} - (1 - \frac{b}{150,000})^{1/2}(200,000)^{1/2}$, as before, an with a lot of algebra you get $b \approx 83704$

(f) Now take the approach of Cumulative Prospective Utility and rewrite the maximization problem from (e) in this framework.
$PU = v(x_1) + \pi(P_2)(v(x_2) - v(x_1))$
or $-\sqrt{200,000} + (\frac{-b}{150,000})^2(-\sqrt{b} - (-\sqrt{200,000}))$

2. Alex is debating how to split up \$100 between a safe and risky asset. Let α be the fraction invested in the safe asset. Assume that the safe asset will pay a gross return rate of 1.05, while the risky asset has the following possible gross returns rates $\begin{cases} -1 & w.p.\ 0.1 \\ 0 & w.p.\ 0.1 \\ 1 & w.p.\ 0.2 \\ 2 & w.p.\ 0.6 \end{cases}$ (Note:gross return rate R means that \$m invested now is worth \$mR in a year).

(a) Calculate his four possible final wealth levels $w1 < w2 < w3 < w4$ if he invests 100α in the the safe asset and $100(1-\alpha)$ in the risky asset. (and has no money other than his \$100 investment).
Sol $w_1 = 100(1.05)\alpha + 100(1-\alpha)(-1) = 105\alpha - 100 + 100\alpha = 205\alpha - 100$
$w_2 = 105\alpha$
$w_3 = 100(1.05)\alpha + 100(1-\alpha)(1) = 105\alpha + 100 - 100\alpha = 5\alpha - 100$
$w_4 = 100(1.05)\alpha + 100(1-\alpha)(2) = 105\alpha + 120 - 120\alpha = -15\alpha + 120$

(b) Suppose he sets his reference point at$x = 105$, the amount he could get if he invested everything in the safe asset. Calculate his cumulative prospective utility, PU, from investing 100α in the the safe asset and $100(1-\alpha)$ in the risky asset, assuming gains/losses s relative to the reference point are evaluated by $v(s) = \begin{cases} \sqrt{s} & if\ s > 0 \\ -2\sqrt{-s} & if\ s < 0 \end{cases}$
Subtracting the reference point. $W_1 = 205\alpha - 100 - 105 = 205\alpha - 205$ $W_2 = 105\alpha - 105$, $W_3 = 5\alpha - 5$, $W_4 = -15\alpha + 15$.
Then check if we consider each outcome as gain or as a loss $W_1 = 205\alpha - 205 > 0 \implies \alpha > 1$, so it is a loss $W_2 > 0 \implies \alpha > 1$, loss. $W_3 > 0 \implies \alpha > 1$ so a loss, finally $W_4 > 0 \implies \alpha < 1$ so it is a gain.
Now using the prospect theory equation $-2\sqrt{(1-\alpha)205} - 2\pi(P_2)\left(\sqrt{(1-\alpha)105} + \sqrt{(1-\alpha)205}\right) - 2\pi(P_3)\left(\sqrt{(1-\alpha)5} - \sqrt{(1-\alpha)105}\right) + \pi(P_4)\left(\sqrt{(1-\alpha)15} - 2\sqrt{(1-\alpha)(5)}\right)$.
Note that $P_2 = p_4 + p_3 + p_2 = .9$, $P_3 = p_3 + p_2 = .8$, $P_4 = p_4 = .6$. Factor out $\sqrt{1-\alpha}$ and the result follows

(c) If you did correctly b your expression should rearrange to the following:

$$PU = \sqrt{1-\alpha}\left[-2(1-\pi(.9))\sqrt{205} - 2(\pi(.9) - \pi(.8))\sqrt{105} - 2\sqrt{5}(\pi(.8) - \pi(.6)) + \sqrt{95}\pi(.6))\right]$$

Argue that Alex will optimally invest either all $\alpha = 1$ or none $\alpha = 0$ of his wealth in the safe asset, depending on the sign of the term in square brackets.
If []>0 then the agent wants to make that value as big as possible, this can be acchievend by setting $\alpha = 0$ If[]<0 then the opposite occurs, the agen wants to minimize the loss, this can by achieved ny setting $\alpha = 1$