Econ 455 Discussion Section-Handout 7

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1 Game Theory/Behavioral Game Theory:

1. Five pirates (Antonio, Beatriz, Carla, David and Elvin) have obained 100 gold doblones and have to divide up the loot. Antonio proposes a distribution of the loot. All pirates vote on the proposal. If half or more agree, the loot is divided as proposed by Antonio. If Antonio fails to obtain support of at least half the the crew (including himself), then he will be killed, in which case the pirates start over with Beatriz as the proposer. If she gets half the remaining crew (including herself) to agree, then the loot is divided as she proposes. If not, then she is killed, and Carla then makes the proposal. If Carla's proposal is not agreed on byhalf the remaining crew, including herself, then she is killed. Finally, David makes a proposal if it isn't accepted by half the crew, including himself, he is killed. If everybody but Elvin is killed, he takes all the loot. The pirates are infinitely sophisticated and rational (as all pirates are). When a pirate is indifferent between voting to kill or supporting a proposal, he votes to kill. You cannot split a doblone, so proposals must be whole numbers of doblones. For each of the following questions, solve using backward induction.

   (a) First, assume each pirate cares only about how many doblones she ends up with more the better). What is the maximum number of doblones Antonio can keep without being killed?

      i. When David proposes, he will propose that he keep all of the loot. Since he is automatically half of the remaining crew, he will receive all of the loot. Given this, when Carla proposes, she knows that Elvin is getting zero if he votes to kill her. Therefore, is she offers Elvin 1 doblon and keeps the rest for herself, Elvin will vote for her and she will 99 doblones. Given this, when Beatriz proposes, she knows that if she is killed, David will get nothing (because of Carla's proposal). Therefore, if Beatriz offers 1 doblone David, he will vote for her proposal. Therefore, she will offer 1 doblone to David and keep 99 for herself. Given this, when Antonio proposes, he knows that if he is killed, Carla and Elvin will get nothing. Therefore, he can offer each of them a doblon and they will vote for him. Therefore, Antonio can actually keep a maximum of 98 doblones for himself!

   (b) Now, in a more behavioral approach, lets assume pirates quite like to see their colleagues killed. In particular, getting to see a colleague killed yields each the utility equivalent of x doblones.

      i. What is the maximum Antonio can keep if x = 2?
      When David proposes, he will propose that he keep all of the loot. Since he is automatically half of the remaining crew, he will receive all of the loot. Given this, when Carla proposes, she knows that Elvin is getting just two (additional) utils if he votes to kill her. Therefore, if she offers Elvin 3 doblones and keeps the rest for herself, Elvin will vote for her and she will get 97. Given this, when Beatriz proposes, she knows that if she is killed, David will get just two (additional) utils (because of Carla's proposal and Beatriz's death). Therefore, if Beatriz offers 3 doblones to David, he will vote for her proposal. Therefore, she will offer 3 dobl ones to David and keep 97 for herself. Given this, when Antonio proposes, he knows that if he is killed, Carla and Elvin will get 2 dobl´ones each (in utils). Therefore, he can offer each of them 3 doblones and they will vote for him. Therefore, Antonio can actually keep a maximum of 94 doblones for himself.

   (c) What is the minimum x such that Antonio will definitely be killed, regardless of the offer he makes??

   If we look at this generally, in each case Antonio just has to buy off two people to save his life. The cheapest two to buy off are the two that are going to get screwed by Carla. Since those two each value his death at x, he needs to offer each of them at least x + 1 to preserve his own life. He can only save his life if $2x + 2 \leq 100$, i.e. if $x \leq 49$. Therefore, x = 50 is the minimum x such that he dies regardless of his offer.

2. (Lemons (Akerlof 1970)) Suppose sellers in a used car market know whether their cars are good or bad but buyers do not and the valuations for buyers and sellers for the two types of cars are as follows:

	Good	Bad
Buyer	60	20
Seller	50	10

   (a) Suppose that a fraction α of the cars on the market are bad, and this is public information. Assume sellers accept offers if indifferent. Describe what it would mean for an outcome to be Pareto efficient in this game.

      i. All it entails here is that the buyers end up with the cars, since they value them more. The prices the transactions take place at are irrelevant in terms of Pareto efficiency.

   (b) if $\alpha = 1.2$, what should buyers offer for a car?
   if they offer 50 then all sellers will accept and they their expected utility is $\frac{5}{6}60 + \frac{1}{6}20 - 50 = \frac{10}{3}$ There's no point offering between \$10 and \$50 because seller who would accept $p \in [10, 50)$ would also accept $p = 10$. then the seller of the bad car will agree, so the expected utility is $\frac{5}{6}(0) + \frac{1}{6}(20 - 10) = \frac{5}{3} < \frac{10}{3}$

3. A child chooses an action that affects both his own income and his parent's income. He chooses A $>$ 0 and incomes for child and parent are: $I_C(A) = A - A^2$, $I_P(A) = A - \frac{1}{2}A^2$. What level of action A maximizes the joint income (i.e. the sum of the two incomes above)? Call this A_C^*, What level of action maximizes the joint income (i.e. the sum of the two incomes above)? Call this A_J^* Now suppose that the child does not care about his parent - he only cares about how much money he has. On the other hand, the parent does care about the child, and may choose to transfer an amount of money B to the child - think of this as a bequest. Their utility functions are as follows: $u_C = \sqrt{I_C(A) + B}$ $u_p = \sqrt{I_p(A) - B} + \frac{1}{2}u_C$. The child chooses A before the parent chooses B, and the parent observes A before choosing B. Use backward induction to solve this game. That is, you should first find how the par- ent best responds to the child's choice of A. Then, given this, you should find what A the child chooses. Once you have found the subgame perfect Nash equilibrium (SPNE) through backward induction, comment on how it compares to A_C^* and A_J^*. Finally, try the problem again except consider the case in which the parent cares very little about his son. That is, replace u_p of above with $u'_p = \sqrt{I_P(A) - B} + \frac{1}{10}u_C$, holding everything else constant. How does this change the child's choice of A in the SPNE?