Econ 455 Discussion Section-Handout 8

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1 Cognitive Dissonance:

1. It will either rain, snow or be sunny today. The weather forecast tells you that the probability of each of these events is $r\, s\, (1-r-s)$ respectively. You have to decide what to wear today, a decision that yields payoffs contingent on the weather as follows:

	Rain	Snow	Sun
Coat	10	0	10
Winter Jacket	0	10	10
T-Shirt	0	0	20

(a) For what r, s is

i. The coat optimal?
The coat is optimal when it yields higher expected payoffs than winter jacket and t-shirt, that is $10r + 10(1 – r – s) \geq 10s + 10(1 – r – s)$ and $10r + 10(1 – r – s) \geq 20(1 – r – s)$.
Which simplifies to $r > s$ and $2r > 1 - s$

ii. The winter Jacket optimal?
The winter jacket is optimal when it yields higher expected payoffs than winter jacket and t-shirt, that is $10s + 10(1 – r – s) \geq 10r + 10(1 – r – s)$ and $10s + 10(1 – r – s) \geq 20(1 – r – s)$
That simplifies to$s > r$ and $2s \geq 1 – r$.

iii. the t-shirt optimal?
The t-shirt is optimal when it yields higher expected payoffs than winter jacket and t-shirt, that is $20(1 – r – s) \geq 10r + 10(1 – r – s)$ and $20(1 – r – s) \geq 10s + 10(1 – r – s)$.
That simplifies to $1 – r – s \geq r$ and $1 – r – s \geq s$.

(b) Suppose you have your own subjective beliefs that put probability $\hat{r}, \hat{s}$ and $(1 - \hat{s} - \hat{r})$ on rain, snow and sun respectively. Also suppose that you choose an action to maximize expected utility at beliefs $(\alpha r + (1-\alpha)\hat{r},\ \alpha s + (1-\alpha)\hat{s},\ \alpha(1-r-s) + (1-\alpha)(1-\hat{r}-\hat{s}))$, and you then distort those beliefs to maximize your expected utility, but only insofar as you can do so without making the action you chose suboptimal. Remember that the algorithm for solving these is to first work out what subjective beliefs would justify each action. Then, for each action, find which subjective beliefs (anything so long as they justify the action) maximize the expected utility of that action. Then compare those maximal expected utilities to pick the action.

i. Supposing $\alpha = 1/2$, $r = 1/2$, and $s = 1/4$ find the action and the beliefs $\hat{r}, \hat{s}$ that you choose
He chooses the action that maximizes expected utility at beliefs: $\left(\frac{1}{4} + \frac{1}{2}\hat{r},\ \frac{1}{8} + \frac{1}{2}\hat{s},\ \frac{1}{8} + \frac{1}{2}(1 – \hat{r} – \hat{s})\right)$
Therefore, he chooses . . .
Coat if the following two conditions are met.
The first: $C \succcurlyeq W$
$10\left(\frac{1}{4} + \frac{1}{2}\hat{r}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 – \hat{r} – \hat{s})\right) \geq 10\left(\frac{1}{8} + \frac{1}{2}\hat{s}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 – \hat{r} – \hat{s})\right)$
That simplifies to $\hat{r} \geq \hat{s} - \frac{1}{4}$
The second: $C \succcurlyeq T$
$10\left(\frac{1}{4} + \frac{1}{2}\hat{r}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 – \hat{r} – \hat{s})\right) \geq 20\left(\frac{1}{8} + \frac{1}{2}(1 – \hat{r} – \hat{s})\right)$
so $2\hat{r} \geq \frac{3}{4} - \hat{s}$
Now he will choose winter jacket if the following two conditions are met.

The first $W \succcurlyeq C$ (opposite of $C \succcurlyeq W$ above) $\hat{r} \leq \hat{s} - \frac{1}{4}$
The second $W \succcurlyeq T$
$10\left(\frac{1}{8} + \frac{1}{2}\hat{s}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right) \geq 20\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right)$
so $\hat{s} \geq 1 - \hat{r} - \hat{s}$
Finally he will choose T-shirt if the following two conditions are met
$T \succcurlyeq W$ $\hat{s} \leq 1 - \hat{r} - \hat{s}$ and $T \succcurlyeq C$ $2\hat{r} \leq \frac{3}{4} - \hat{s}$
Now we know the subjective beliefs that justify each action. The second step of the algorithm is to, for each action, find which subjective belief, out of those that justify that action, maximized the expected utility from that action. Formally, for each actions, this amounts to solving a constrained maximization problem over two variables, with the constraints being to ensure that we're only considering subjective beliefs that justify the given action. However, the problem is actually a fair bit simpler than that. Consider the actions one-by-one:
If you wear the coat, you don't care if it rains or if it's sunny, but you don't like when it snows. Therefore, optimal subjective beliefs will be those that set $\hat{s} = 0$ as for $\hat{r}$you're indifferent as to whether it's rainy or sunny, so any $\hat{r} \in [0, 1]$ is optimal. However, we also need that $\hat{r}$ to be such that $\hat{r}$, $\hat{s}$ justify wearing the coat. $2\hat{r} \geq \frac{3}{4} - \hat{s}$, so $\hat{r} = 1$
Therefore $EU = 10\left(\frac{1}{4} + \frac{1}{2}\hat{r}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right) = \frac{35}{4}$
If you wear the winter jacket, you don't care if it snows or if it's sunny, but you don't like when it rains. Therefore, optimal subjective beliefs will be those that set $\hat{r} = 0$. As for $\hat{s} = 1$, you're indifferent as to whether it's snowy or sunny, so any $\hat{s} \in [0, 1]$ is optimal. However, we also need that $\hat{s}$ to be such that $\hat{r}$,$\hat{s}$ justify wearing the coat. In particular, that means that $\hat{s} \geq 1 - \hat{r} - \hat{s}$, thus $\hat{s} = 1$
then $EU = 10\left(\frac{1}{8} + \frac{1}{2}\hat{s}\right) + 10\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right) = \frac{15}{2}$.
Now if you wear the t-shirt, you only get utility if it's sunny. Therefore, optimal subjective beliefs will be those that set $\hat{r} = 0$ and $\hat{s} = 0$
so $EU = 20\left(\frac{1}{8} + \frac{1}{2}(1 - \hat{r} - \hat{s})\right) = \frac{25}{2}$, since $\frac{25}{3}$ is the largest of the three, the optimal action is to wear the t-shirt and the optimal beliefs are $\hat{s}, \hat{r} = 0$

ii. Supposing $\alpha = 1$, $r = 1/2$, $s = 1/4$ find the action the beliefs $\hat{r}$, $\hat{s}$ that you choose
When $\alpha = 1$, we simply take the objective probabilities as given, so we return to the case in (a) with $r = \frac{1}{2}$ and $s = \frac{1}{4}$ the coat conditions are satisfied so we wear the coat.

2. You are interested in applying to grad school. To get into grad school you need to present the GRE and get a decent score. The GRE is an ability test and you ignore your ability level. However you know that your true abilityθ is one of the four GPAs you got from college divided by four. The first year of college you got 3.5, on the second year you got 3.0, on the third year you got 4.0 and on the fourth and last year you got 3.5.

   At $t = 0$ you can go and ask your TA about your true ability, he knows you well.

   At $t = 1$ you must decide whether to take or not the GRE. You will get the score you need with probability θ and fail with probability $1 - \theta$. If you do not take the GRE you will not go to grad school. Taking the GRE has a utility cost of 1, since it is a horrible test.

   At period 2, your will know if you got the required score to apply to grad school, which gives you a benefit of 5 if you succeed and 0 if you fail. Suppose also that you are a quasihyperbolic discounter. with $\delta = 1$ and $\beta = 1/4$. For simplicity assume you are risk neutral.

   (a) Solve by backwards induction, at $t = 1$ will you take the GRE?
   At period 1 if you do not take the GRE $EU = 0$, however if you take the GRE $EU = -c + \beta\delta E[\theta]V = -1 + \frac{1}{4}(1)E[\theta]5$.
   Therefore we need to find $E[\theta]$, but before that we need to know the probability mass function (recall for discrete random variables we say PMF instead of PDF). Bu to do so first we need to know the possible values θ can take. From your GPA $\theta = \{3.5/4, 3/4, 4/4, 3.5/4\}$ in other words the PMF

   $$f(\theta) = \begin{cases} 1/4 & if\, \theta = 3/4 \\ 1/2 & if\, \theta = 7/8 \\ 1/4 & if\, \theta = 1 \end{cases}$$

   Hence $E[\theta] = \sum_{\theta=3/4}^{1} \theta f(\theta) = \frac{3}{4} \times \frac{1}{4} + \frac{7}{8} \times \frac{1}{2} + 1 \times \frac{1}{4} = \frac{3}{16} + \frac{7}{16} + \frac{4}{16} = \frac{7}{8}$.
   So $EU = -c + \beta\delta E[\theta]V = -1 + \frac{1}{4}(1)(\frac{7}{8})5 = \frac{3}{32}$, since $\frac{3}{32} > 0$ then you should take the GRE.

(b) At $t = 0$ What would you do if you decided not talk to your TA about your true ability?

If you do not talk with your TA then if you do not take the GRE $EU = 0$.

If you take the GRE $EU = -\beta\delta c + \beta\delta^2 E[\theta]V = -(\frac{1}{4})(1)(1) + (\frac{1}{4})(1)(\frac{7}{8})(5) = \frac{35-8}{32} = \frac{27}{32}$, again since $\frac{27}{32} > 0$ you take the GRE

(c) Write down the possible payoff from period 0 perspective if you talk to your TA.

If you talk to your TA, then at period 1 you will know your true θ.

That means that if you will take the GRE at period one only if $EU = -c + \beta\delta\theta V > 0$ i.e. your true ability should be $\theta > \frac{c}{\beta\delta V}$

Hence at period zero you know that , $EU = 0Pr(\theta \leq \frac{c}{\beta\delta V}) + Pr(\theta > \frac{c}{\beta\delta V}) \left\{-c\beta\delta + \beta\delta^2 E\left[\theta|\theta > \frac{c}{\delta\beta V}\right] V\right\}$.

We can simplify by $EU = Pr(\theta > \frac{4}{5}) \left\{-\frac{1}{4} + \frac{5}{4} E\left[\theta|\theta > \frac{4}{5}\right]\right\}$, therefore we need to find $Pr(\theta > \frac{4}{5})$ and $E\left[\theta|\theta > \frac{4}{5}\right]$

So $Pr(\theta > \frac{4}{5}) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ and $E\left[\theta|\theta > \frac{4}{5}\right] = \frac{\sum_{\theta=7/8}^{1} \theta f(\theta)}{Pr(\theta > \frac{4}{5})} = \frac{\frac{7}{8} \times \frac{1}{2} + 1 \times \frac{1}{4}}{\frac{3}{4}} = \frac{\frac{11}{16}}{\frac{12}{16}} = \frac{11}{12}$.

Hence $EU = \frac{3}{4}\left(-\frac{1}{4} + \frac{5}{4}(\frac{11}{12})\right) = \frac{129}{196}$

(d) Will you talk to your TA?

No since $\frac{27}{32} > \frac{129}{196}$