Econ 455 Discussion Section-Handout 9

Emilio Cuilty

04/15/2016

---

1 Cognitive Dissonance II/Bounded Rationality:

1. It will either rain, snow or be sunny today. The weather forecast tells you that the probability of each of these events is $r\, s\, (1-r-s)$ respectively. You have to decide what to wear today, a decision that yields payoffs contingent on the weather as follows:

	Rain	Snow	Sun
Coat	10	0	10
Winter Jacket	0	10	10
T-Shirt	0	0	20

   (a) For what r, s is

      i. The coat optimal?

      ii. The winter Jacket optimal?

      iii. the t-shirt optimal?

   (b) Suppose you have your own subjective beliefs that put probability $\hat{r}, \hat{s}$ and $(1-\hat{s}-\hat{r})$ on rain, snow and sun respectively. Also suppose that you choose an action to maximize expected utility at beliefs $(\alpha r + (1-\alpha)\hat{r},\ \alpha s + (1-\alpha)\hat{s},\ \alpha(1-r-s) + (1-\alpha)(1-\hat{r}-\hat{s}))$, and you then distort those beliefs to maximize your expected utility, but only insofar as you can do so without making the action you chose suboptimal. Remember that the algorithm for solving these is to first work out what subjective beliefs would justify each action. Then, for each action, find which subjective beliefs (anything so long as they justify the action) maximize the expected utility of that action. Then compare those maximal expected utilities to pick the action.

      i. Supposing $\alpha = 1/2$, $r = 1/2$, and $s = 1/4$ find the action and the beliefs $\hat{r}, \hat{s}$ that you choose

      ii. Supposing $\alpha = 1$, $r = 1/2$, $s = 1/4$ find the action the beliefs $\hat{r}$, $\hat{s}$ that you choose

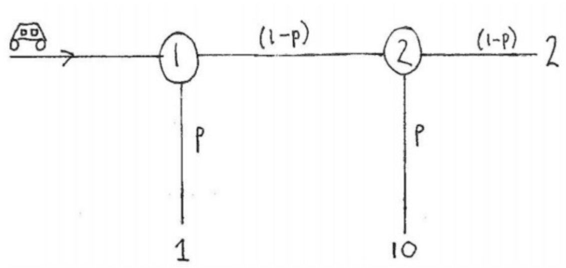
2. Find the mixed Nash equilibrium in the following game. Note: We're doing this to to see how similar it is to finding an interim optimal mixed strategy for a forgetful driver, which we'll do next. Hint: To be willing to mix across actions, a player must be indifferent between those two actions – exploiting this allows us to solve for the mixed Nash equilibrium.

	B	D
A	1,0	0,3
C	0,2	4,0

   You can first underline best responses and convince yourself that no pure-strategy Nash equilibrium exists.

   In terms of finding the mixed stategy NE, let's suppose P1 is putting weight α on A and $(1-\alpha)$ on C while P2 putting weight β on B and $(1-\beta)$ on D. Then the mixed strategy forP2, P1 must be indifferent between playing A and C. That is $U_1(A, \beta B + (1-\beta)D) = U_1(C, \beta B + (1-\beta)D)$, $(1)\beta + (0)(1-\beta) = \beta(0) + (1-\beta)4$, $\beta = 4(1-\beta)$ so $\beta = \frac{4}{5}$. We can do the same for Player 1. $U_2(B, \alpha A + (1-\alpha)C) = U_2(D, A\alpha + (1-\alpha)C)$, hence $\alpha 0 + 2(1-\alpha) = \alpha 3 + 0(1-\alpha)$, so $\alpha = \frac{2}{5}$. Hence we conclude that the NE is $\left\{\left(\frac{2}{5}, \frac{3}{5}\right), \left(\frac{4}{5}, \frac{1}{5}\right)\right\}$

3. Consider a forgetful driver (doesn't know which intersection he is at) whose payoffs are as described in the picture below:



(a) Find the ex ante optimal pure strategy ($i.e. p = 1\, or\, p = 0$).

Remember that ex ante means that he's deciding this probability before he sets off on his drive. If $p = 1$, he will get a payoff of 1. If $p = 0$ he will get a payoff of 2, therefore $p = 0$ is the optimal pure stratgy.

(b) Find the ex ante optimal mixed strategy.

He want to maximize $1p + 10(1-p)p + 2(1-p)^2$, then $F.O.C.$ we have that $1 + 10(1-2p) - 4(1-p) = 0$, $p = \frac{7}{16}$

(c) Is$p = 1$ interim optimal? Show why or why not.

Remember that the interim concerns how we feel about our plan once we actually show up at an intersection (could be either the first or the second). If we know our strategy is that $p = 1$ then as we show up at an intersection we know it is the first intersection, because we would never make it past the first intersection. Given that, and given that we assume we're going to follow our plan in the future, we have an incentive to deviate and go straight rather than turn, because then we'll get 10 instead of 1. (because $p = 1$ implies we'll definitely turn right at the next intersection). Therefore $p = 1$ is not interim optimal.

(d) Is $p = 0$ interim optimal? Show why or why not.

If $p = 0$, then our path always takes us through both intersections, so when we show up at an intersection, we think it is equally likely that we're at either of the two intersections. Then, if we choose to deviate (by turning right), we'll get 1 with a probability of a half and 10 with a probability of a half, giving us an expected payoff of 11/2. Since $11/2 > 2$, we do have an incentive to deviate from our plan and turn. Therefore$p = 0$ is not interim optimal.

(e) Find an interim optimal $p \in (0, 1)$. Compare it to the ex ante optimal mixed strategy.

To find an interim optimal $p \in (0, 1)$ is much the same as finding a mixed Nash equilibrium. Our strategy needs to be such that, given our strategy, we're indifferent between turning right and going straight. Note that if we're turning right with probability p, the probability we're at the first and second intersection respectively is as follows: $\mu(h_1|X) = \frac{Pr^\sigma(h_1)}{Pr^\sigma(h_1)+Pr^\sigma(h_2)} = \frac{1}{1+(1-p)} = \frac{1}{2-p}$. $\mu(h_2|X) = \frac{1-p}{1+(1-p)} = \frac{1-p}{2-p}$. His expected payoffs EU from the two actions. If he is truly willing to follow this mixed strategy (i.e. $p \in (0, 1)$) then these must be equal, so let's set them equal: $EU(straight|p) = EU(right|p)$, $\mu(h_1|X)(10p + 2(1-p)) + \mu(h_2|X)(2) = \mu(h_1|X)(1) + \mu(h_2|X)(10)$ so $\frac{1}{2-p}(10p + 2(1-p)) + \frac{1-p}{2-p}(2) = \frac{1}{2-p}(1) + \frac{1-p}{2-p}10)$ so $p = \frac{7}{16}$