Discussion Handout #13 Solutions

Oligopoly Models

Finding a Cournot-Nash Duopoly Equilibrium

The Cournot model of oligopoly is used to model industries where there are a few firms that compete in the quantity of output that they produce. We usually assume that all the firms are similar or identical to each other, and that the product they produce is homogeneous. Since the total market quantity is the sum of the quantities that each firm produces, the equilibrium price depends on the actions of every firm. Hence every firm's profit depends on the actions of every other firm! To maximize profits, firms must therefore act strategically - that is, they must take other firms' actions into account when determining what to produce. It is as if firms are *reacting* to each other, and much of this model boils down to finding *reaction functions* for the firms.

Although originally pioneered by Augustin Cournot, this model is also a very early model in the field of Game Theory - a field which really took off after John Nash's discoveries in the 1950s. (This the John Nash of *A Beautiful Mind* fame.) We often refer to equilibria of this model as Cournot-Nash equilibria.

The following are steps to find a Cournot-Nash equilibrium with two firms (A and B) with identical, constant marginal costs and linear demand.

1. Find Q_{PC}, the market quantity under perfect competition.

2. Find Q_M, the quantity that would be produced by a monopoly.

3. Set up reaction function axes in $q_B \times q_A$ space.

4. Think. If B produces $q_B = 0$, then firm A is a monopolist, so it should produce $q_A = Q_M$. Plot this point.

5. Think. If B produces Q_{PC}, then if A produces anything, it will make a loss. So in this case $q_A = 0$. Plot this point and connect the dots. This is firm A's reaction function, $q_A = f(q_B)$.

6. Plot $B's$ reaction function symmetrically, $q_B = g(q_A)$.

7. The intersection of the reaction functions is the Cournot-Nash equilibrium (q_A^*, q_B^*). Add these to find the Cournot market quantity Q_C and price. **Trick: If firms have identical, constant marginal costs and face linear demand, then $q_A^* = q_B^* = \frac{2}{3}Q_M$.**

8. (If necessary): Use your graph to determine equations for the firms' reaction functions. Under the assumptions we have made, these will be linear: $q_A = Q_M - \frac{1}{2}q_B$ and $q_B = Q_M - \frac{1}{2}q_A$

Cournot Duopoly Example

1. Consider the market for central processing units (CPUs), a key component in modern computers. This market consists of two firms: Intel and AMD. For simplicity, assume that both Intel and AMD have identical cost structures, where $MC = AC = 30$ for each firm. On any given day, the market demand for CPUs is given by $P = 120 - Q$.

   (a) Suppose the market for CPUs was controlled by a monopoly with the same cost structure as Intel and ADM. How many CPUs would this monopoly produce (call this Q_M), and what price would it charge P_M? *Solving the monopolist's problem in the usual way yields* $Q_M = 45$, $P_M = \$75$

   (b) Suppose instead the market for CPUs was perfectly competitive, with every firm having the same cost structure as Intel and AMD. What would be the market equilibrium quantity Q_{PC} and price P_{PC}? *Solving the perfectly competitive firm's problem yields* $Q_{PC} = 90$ *and* $P_{PC} = \$30$.

   (c) Now return to reality, where Intel and AMD compete as Cournot duopolists. What is the reaction function of Intel, $q_{Intel} = f(q_{AMD})$? What is the reaction function of AMD, $q_{AMD} = g(q_{Intel})$? *Using the procedure outlined above, one can see that the reaction function of Intel is* $q_{Intel} = 45 - \frac{1}{2}q_{AMD}$ *and* $q_{AMD} = 45 - \frac{1}{2}q_{Intel}$.

   (d) Find the quantity produced by each firm in a Cournot-Nash equilibrium, q^*_{Intel} and q^*_{AMD}. Then find the market quantity Q_C and market price P_C under this Cournot duopoly. *Either by finding the intersection of the frims' reaction functions or by using* ***Trick,*** *we can see that* $q^*_{Intel} = q^*_{AMD} = \frac{2}{3}Q_M = \frac{2}{3} \cdot 45 = 30$. *Therefore* $Q_C = 60$ *and* $P_C = 60$.

   (e) Compare the three industrial structures: monopoly, Cournot duopoly, and perfect competition. Rank these in terms of firm profits and the welfare of consumers (Hint: there is no need to calculate anything here. Use your intuition to rank these by comparing prices and quantities only.) *In terms of firm profits, profit for monopoly > profit for a firm in Cournot > profit for a perfectly competitive firm = 0 since marginal cost equals average cost and is constant. In terms of consumer surplus, CS under perfect competition > CS under Cournot > CS under monopoly.*

Finding a Price Leadership Equilibrium

In the Price Leadership (sometimes called Dominant Firm) model of oligopoly, there is assumed to be a large, dominant firm with low marginal costs and many small, identical, "fringe" firms with higher costs. The dominant firm takes the behavior of the small firms as given, meaning the demand curve it faces is the market demand NET of the supply of the small firms. Thus the demand curve faced by the dominant firm is really a "residual" demand curve. Since this residual demand curve is downward-sloping, the dominant firm acts like a monopolist with respect to this residual demand to find the profit-maximizing quantity and the market price. The small firms then take this price as given and produce as if they were in a perfectly competitive market. Most of the difficulties in this model arise when finding the residual demand curve.

The following are steps to find an equilibrium in the Price Leadership model of oligopoly assuming a linear market demand curve and constant or linear marginal costs.

1. Find the total supply of the small firms $Q_{SF}(P)$ if it is not already given to you. Be sure to add quantities, NOT prices!
2. Find the residual demand curve that is faced by the dominant firm and call it $Q_D(P)$. This will necessarily be a curve with TWO kinks.
   (a) The rightmost kink will be at the price for which the small firms supply nothing: where $Q_{SF}(P) = 0$. Below this price, the residual demand is identical to market demand.
   (b) The leftmost kink point occurs at the price such that the supply of the small firms is equal to market demand: $Q_{SF}(P) = Q_M(P)$, where Q_M is the market demand curve. Above this price, the dominant firm's residual demand is zero: $Q_D(P) = 0$.
   (c) Between these two kink points, the dominant firm's residual demand is found by subtracting the supply of the small firms from the market demand. That is, $Q_D(P) = Q_M(P) - Q_{SF}(P)$. **Note: it usually suffices to only compute this portion of the residual demand if you want to save some time.**
3. Using the portion of the dominant firm's residual demand found in part 2(c), compute the dominant firm's marginal revenue curve MR_D in the usual way (i.e., the same as you would with a monopoly).
4. The dominant firm acts like a monopoly with respect to its residual demand. Thus, to find the quantity Q_D^* it produces, set $MR_D = MC_D$. Plug this Q_D^* back into the residual demand curve to find the market price P^*.
5. To find Q_{SF}^*, plug P^* into $Q_{SF}(P)$. Use an analogous procedure for market quantity Q_M^*.
6. Verify that $Q_M^* = Q_D^* + Q_{SF}^*$. If these are not equal, then you made a mistake somewhere.

Price Leadership Example

1. Consider the price leadership model of oligopoly in the market for tablet computers. Suppose the market demand for tablets is given by

$$P = 100 - \frac{1}{10} Q_M.$$

Suppose the dominant firm, Apple, has a marginal cost function MC_D given by

$$MC_D = \frac{9}{10} Q_D + 2.5.$$

Furthermore, suppose there are 20 other identical small firms that produce tablets, EACH with marginal cost function MC_{SF} given by

$$MC_{SF} = 2q_{SF} + 5.$$

(a) Find the supply curve for small firms as a function of price, $Q_{SF}(P)$. *First, one needs to invert MC_{SF} to find each firm's supply curve. Doing so yields $q_{SF} = \frac{P-5}{2}$. Since there are 20 firms, we can add quantities by multiplying by 20, resulting in $Q_{SF}(P) = 10P - 50$.*

(b) Find the residual demand function for the dominant firm, $Q_D(P)$. (Make sure you find all 3 segments of this curve, yielding a piecewise demand function. The easiest way to do this is to find the two kink points.) *From our equation for Q_{SF}, we can see that $Q_{SF} = 0$ only when $P = \$5$. At prices below 5, residual demand is just market demand. The second kink point is at the price where the small firm supply equals the market demand. Solving for this price yields $P = \$52.50$. So above this price, the quantity supplied by the dominant firm is 0. If price is between \$5 and \$52.50, then to find the dominant firm's residual demand we subtract the small firms' supply from market demand. That is, $Q_D = Q_M - Q_{SF} = (1000-10P)-(10P-50) = 1050-20P$. In summary,*

$$Q_D(P) = \begin{cases} 1000 - 10P & \textit{if } P < 5 \\ 1050 - 20P & \textit{if } 5 \leq P \leq 52.50 \\ 0 & \textit{if } P > 52.50 \end{cases}$$

(c) Solve for the quantity produced by the dominant firm in equilibrium, Q_D^*. *The dominant firm acts like a monopoly with respect to its residual demand. So first we must find its marginal revenue curve MR_D. We can assume that he will produce a quantity that yields a price between \$5 and \$52.50. Inverting the relevant portion of the residual demand curve yields $P = 52.50 - \frac{1}{20}Q_D$, which implies $MR_D = 52.50 - \frac{1}{10}Q_D$. Setting $MR_D = MC_D$ yields $Q_D^* = 50$.*

(d) Find the equilibrium price P^*. *Plugging $Q_D^* = 50$ into the dominant firm's residual demand curve in the relevant region yields $P^* = \$50$.*

(e) Find the quantity produced by the small firms in equilibrium Q_{SF}^* and the market quantity in equilibrium Q_M^*. *Plugging $P^* = 50$ into the small firms' supply curve yields $Q_{SF}^* = 450$, and plugging $P^* = 50$ into the market demand curve yields $Q_M^* = 500$. One can easily verify that $Q_M^* = Q_D^* + Q_{SF}^*$.*

Collusion Example

1. Consider the model of oligopoly given in the **Cournot Duopoly Example** on this handout.

(a) Instead of competing as Cournot duopolists, suppose Intel and AMD decide to collude. They jointly decide what to produce and then split the profits evenly between them. How much do they each produce in this scenario? How much profit does each earn? *Solving this problem in the same way as one would for a monopoly, we find that together they produce $Q = 45$, and hence each make $q = 22.5$ units. The price is therefore $P = \$75$, yielding profits of $\pi = 22.5(75 - 30) = \$1012.50$ per firm.*

(b) Suppose Intel decides to cheat in secret and produce an extra 5 CPUs. What are the profits of Intel and AMD in this scenario? Does it pay to be the cheater? *Since*

Intel does this in secret, AMD does not have a chance to react strategically. Thus the total market quantity is just $45+5=50$. *This yields a price of* $P=\$70$. *Since AMD is still producing* $q=22.5$, *it earns profits* $\pi_{AMD}=22.5(70-30)=\$900$. *However, Intel earns profits* $\pi_{Intel}=27.5(70-30)=\$1100$. *Since Intel's profits are higher, it does pay to be the cheater.*

(c) Suppose in addition to Intel cheating and producing an extra 5 CPUs, AMD also decides to cheat in secret and produce an extra 5 CPUs. Are the firms better or worse of compared to when they colluded in (a)? (Note: we will see more examples of problems like this when we study game theory.) *Since they are both cheating in secret, we assume they are not acting strategically. The total market quantity is therefore* $45+10=55$. *This yields a price of* $P=\$65$. *They both earn profits of* $\pi=27.5(65-30)=\$962.50$. *In this case, since both firms are cheating, they are both worse off. We will see more applications of problems like this when we study game theory.*