Discussion Handout #4 Solutions

Elasticity

1. Consider Doru's demand for fleece jackets. His demand schedule is given below, along with the slope of his demand curve at each point. Using the point elasticity technique, calculate Doru's price elasticity of demand to complete the table. Then answer the questions below.

Slope	Q	P	Price elasticity of demand, ϵ_D
$-8/5$	5	4	$\epsilon_D = -\frac{5}{8} \cdot \frac{4}{5} = -\frac{1}{2}$
$-27/5$	$10/3$	9	$\epsilon_D = -\frac{5}{27} \cdot \frac{3 \cdot 9}{10} = -\frac{1}{2}$
$-64/5$	$5/2$	16	$\epsilon_D = -\frac{5}{64} \cdot \frac{2 \cdot 16}{5} = -\frac{1}{2}$
-25	2	25	$\epsilon_D = -\frac{1}{25} \cdot \frac{25}{2} = -\frac{1}{2}$

   (a) Does Doru's demand curve exhibit the law of demand? Why or why not? *Yes. The law of demand holds whenever quantity demanded decreases as price increases, which is the case here.*

   (b) Is Doru's demand curve linear? Why or why not? Give TWO reasons. *No, Doru's demand curve is not linear. First of all, since the slope of Doru's demand is not constant, his demand cannot be linear. Secondly, (and more importantly for our study of elasticity), notice that Doru's price elasticity of demand is exactly the same at every point. We know that price elasticity of demand varies along a linear demand curve, which is not the case here.*

   (c) At any given price, is Doru's demand for fleece jackets elastic or inelastic? *Since his price elasticity of demand is -0.5, (which is between -1 and 0), his demand for fleece jackets is inelastic. (An easy way to remember what numbers are elastic/inelastic is as follows: Since 0 is a fixed number that does not "stretch,"elasticities close to 0 are inelastic. Since 1 is one unit, we have unitary elasticity at -1. And since infinity is a number that is not fixed (i.e., it can be "stretched"), as we approach negative infinity, demand becomes more elastic.)*

   (d) Interpret Doru's price elasticity of demand for fleece jackets in the jargon of percentage changes. *Recall that*

$$\epsilon_D = \frac{\%\Delta Q}{\%\Delta P}$$

   Since Doru's price elasticity of demand for fleece jackets is -0.5, a 2% increase in the price of fleece jackets will lead to a 1% decrease in the quantity of fleece jackets Doru demands. For interested students, Doru's demand function is actually

$$Q = \frac{10}{\sqrt{P}}.$$

This function is an example of a constant elasticity demand curve (price elasticity of demand is the same at every point). In general, constant elasticity demand curves are of the form

$$Q = bP^{(-a)}$$

where a and b are positive real numbers and –a is the price elasticity of demand.

2. Consider the demand for issues of *The Economist* magazine. Suppose the price of one copy of *The Economist* is \$5.00, and at this price, 1000 copies are sold. Then, suppose an increase in the price of ink causes the price per copy of *The Economist* to increase to \$5.50.

   (a) If 800 copies are sold at the new price of \$5.50, what is the price elasticity of demand? Use the midpoint formula. *The midpoint formula is*

   $$\epsilon_D = \frac{\frac{Q_2 - Q_1}{Q_2 + Q_1}}{\frac{P_2 - P_1}{P_2 + P_1}}.$$

   So we have

   $$\epsilon_D = \frac{\frac{800-1000}{800+1000}}{\frac{5.50-5}{5.50+5}} = \frac{-\frac{200}{1800}}{\frac{.5}{9.5}} = \frac{-\frac{1}{9}}{\frac{1}{21}} = -\frac{7}{3}.$$

   (b) Is demand for *The Economist* elastic or inelastic between \$5.00 and \$5.50 (i.e., at the midpoint of these you found in (a))? *Since* $-\frac{7}{3} < -1$*, demand is elastic in this region.*

3. Consider Joe's demand for Star Wars books. At a price of \$8 per book, Joe buys 4 Star Wars books, and at this price, his price elasticity of demand is $\epsilon_D = -2$. Assuming Joe's demand for Star Wars books is linear, find his demand equation. *Using the point elasticity technique, we have*

   $$-2 = \frac{1}{slope} \cdot \frac{8}{4}.$$

   We can see that slope = -1. Since Joe's demand curve is linear, we have

   $$P = b - 1Q.$$

   where b is the P-intercept of the demand curve. Plugging in $P = 8$ *and* $Q = 4$*, we have*

   $$8 = b - 4,$$

   so $b = 12$*. Joe's demand for Star Wars books is thus*

   $$P = 12 - Q$$

4. Consider the market for bicycles. Assume the demand for bicycles is linear, with a slope of -0.5.

(a) Suppose at a price of \$300, we observe that 700 bicycles are sold. Using the point elasticity technique, determine if this equilibrium is above or below the midpoint of the demand curve. *The point elasticity formula is*

$$\epsilon_D = \frac{1}{slope} \cdot \frac{P}{Q}.$$

We then have

$$\epsilon_D = \frac{1}{-\frac{1}{2}} \cdot \frac{300}{700} = -\frac{6}{7}.$$

Since $-\frac{6}{7} > -1$, we know that we are on the inelastic portion of the linear demand curve. That is, we are below the midpoint.

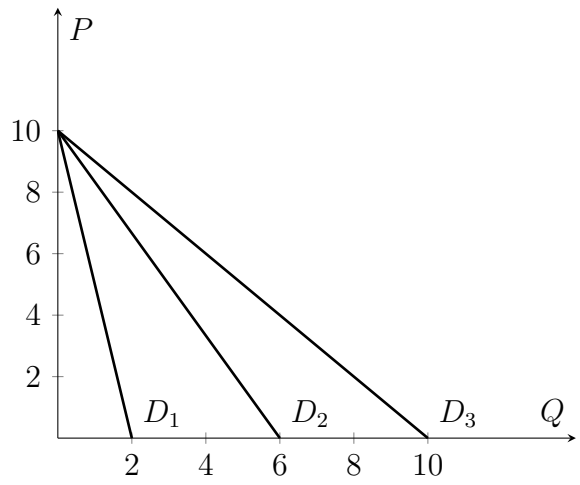
(b) Suppose Trek (a bicycle manufacturer headquartered in Wisconsin) sells all the bicycles in this market. If Trek raised the price of its bicycles above \$300, would Trek increase its total revenue? Why or why not? *At P=300 and Q=700, raising the price of bicycles would increase Trek's total revenue. Recall that total revenue $TR = P \cdot Q$. Since demand is inelastic at this point (-6/7 specifically), a 1% increase in price will decrease quantity by less than 1% (6/7% specifically). Hence, quantity demanded will not respond much to small increases in price and therefore total revenue will increase.*

(c) Suppose we now observe that 500 bicycles are sold at a price of \$400 each. If Trek were to increase the price of bicycles above \$400, would its total revenue increase? Why or why not? *The point elasticity technique yields*

$$\epsilon_D = \frac{1}{-\frac{1}{2}} \cdot \frac{400}{500} = -\frac{8}{5}.$$

At this point, demand is elastic, hence we are above the midpoint of the demand curve. In this case, raising the price of bicycles would DECREASE Trek's total revenue. A 1% increase in price would decrease quantity demanded by more than 1% (specifically, quantity demanded would decrease by 8/5%). Hence, quantity demanded is very responsive to small changes in price. If Trek increased the price of bicycles above \$400, quantity demanded would decrease by a lot, and hence total revenue would decrease. NOTE: Another useful thing to know is that total revenue is maximized along a linear demand curve at the point where the price elasticity of demand is -1, i.e., where demand is unit elastic. Since we know that demand is unit elastic at the midpoint of the curve, we know that total revenue is maximized at the midpoint of a linear demand curve. As we move towards the midpoint, total revenue increases, and as we move away from it, total revenue decreases.

5. In the plot below, which demand curve is the most price elastic at $P = 5$? $D_1, D_2,$ or D_3?



This is a trick question. Notice that $P = 5$ is the midpoint of all three demand curves. Hence $\epsilon_D = -1$ at $P = 5$ for all three curves, and no curve is more price elastic than any other at $P = 5$. (Note that these curves have the same price elasticity of demand at EVERY price! This is because they all have the same P-intercept!)

6. Suppose the demand for Whatchamacallits (a type of candy bar) is given by

$$P = 4 - (e^{-3}\sin(\frac{5}{6})\sqrt{2})Q.$$

What is the price elasticity of demand for Whatchamacallits at $P = 2$? *Despite how ugly the coefficient on Q looks, it is just a constant. Thus this demand curve is just a linear function - nothing fancy at all! Note also that $P = 2$ is the midpoint of this curve since the P-intercept is 4. Thus, as at any midpoint, the price elasticity of demand at $P = 2$ is $\epsilon_D = -1$.*

7. Suppose when the price of good X increases, the quantity demanded of good Y increases. Are goods X and Y complements, substitutes, or neither? What might be the cross-price elasticity of demand for goods X and Y? Give an example of two such goods X and Y that behave in this way. *X and Y are substitutes: Since the price of X increases, people demand fewer X and more Y. That is, they will substitute their consumption away from X and towards Y. The cross-price elasticity of demand for substitutes is positive. (Think about this in terms of percentage changes if this is not clear). So possible cross-price elasticities include 2 or 5, for example. Examples of goods that are substitutes in consumption are pens and pencils or apples and oranges.*

8. Suppose when the price of good U decreases, the quantity demanded of good V increases. Are goods U and V complements, substitutes, or neither? What might be the cross-price elasticity of demand for goods U and V? Give an example of two such goods U and V that behave in this way. Give an example of two such goods U and V that behave in this way. *U and V are complements: since the price of U decreases, people will demand more of both U and V. In other words, U and V are goods that are consumed together. The price elasticity of demand for complements is negative. (Think*

about this in terms of percentage changes if this is not clear). So possible cross-price elasticities include -1 or -3, for example. Examples of complements are tennis rackets and tennis balls or hot dogs and hot dog buns.

9. Suppose when the price of good A increases, the quantity demanded of good B does not change. What is the cross-price elasticity of demand for goods A and B? *The quantity demanded of good B does not respond to a change in the price of A. So, A and B are unrelated in consumption, and their cross-price elasticity of demand is exactly 0.*

10. Suppose when Nick's income increases, Nick buys more golf balls than he did before. Are golf balls a normal or inferior good for Nick? What might be Nick's income elasticity of demand for golf balls? *Nick's consumption of golf balls increases when his income increases. So golf balls are a normal good for Nick. Normal goods have positive income elasticity of demand. (If this is not clear, think about it in terms of percentage changes). So possible income elasticities of Nick's demand for golf balls include 1, 5, or 8, for example.*

11. Now consider Nick's demand for Mountain Dew. When his income increases, the quantity of Mountain Dew he demands does not change. What is Nick's income elasticity of demand for Mountain Dew? *Nick's consumption of Mountain Dew does not depend on his income. This means his income elasticity of demand for Mountain Dew is exactly 0.*