Running head: PROBLEM SET 1

Problem Set 1

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Problem 1. Show that

$\{X_i\}_{i=0}^{T}$ is a white noise process where $X_i \sim N(0, \sigma^2)$ and X_is are independent

Problem 2. Let A_0 denote the value of A in period 0, and let the behavior of lnA be given by $lnA_t = \bar{A} + g_t + \widetilde{A_t}$ and $\widetilde{A_t} = \rho_A \widetilde{A_{t-1}} + \epsilon_{A,t}$ with $-1 < \rho_A < 1$.

- Express lnA_1,lnA_2 and lnA_3 in terms of lnA_0, $\epsilon_{A,1}$, $\epsilon_{A,2}$, $\epsilon_{A,3}$, $\bar{A}$ and g
- In the light of the fact that the expectations of the $\epsilon_A s$ are zero, what are the expectations of lnA_1,lnA_2 and lnA_3 given $lnA_0\bar{A}$ and g

Problem 3. Suppose the period t utility function, u_t is $u_t = lnc_t + b(1 - l_t)^{1-\gamma}/(1 - \gamma)$, $b > 0$,$\gamma > 0$, instead of the utility function used by Romer.

- Consider the one period problem analogous to that investigated in class, How, if at all, does labor supply depend on the wage?
- Consider the two period problem analogous to that investigated in class,how does the relative demand for leisure in the two periods depend on the relative wage? How does it depend on the interest rate? explain why γ affects the responsiveness of labor supply to wages and interest rate

Problem 4. Consider the two period problem investigated in class. use the Romer's utility function

- Show that an increase in both w_1 and w_2 that leaves $\frac{w_1}{w_2}$ unchanged does not affect l_1 or l_2
- Now assume that the house hold has initial wealth of $Z > 0$. does the former result is preserved?

Problem 5. Consider the function $f(k)$ defined in the class of Introduction to Dynamic Programming within the model of growth. Given the properties of the function $F(k, l)$ proof that:

- $f(k)$ is continuously differentiable,
- Strictly increasing
- Strictly concave with:

- $f(0) = 0$
- $f'(k) > 0$

- $lim_{k \to 0} f'(k) = \infty$
- $lim_{k \to \infty} f'(k) = 1 - \delta$

Problem 6. Show that in the finite scenario the deterministic model of growth leads to the following Kuhn/Tucker and slackness conditions (equations 5 and 6)

- $\beta f'(k_t) U'(f(k_t) - k_{t+1}) = U'(f(k_{t-1}) - k_t)$
- $k_{T+1} = 0$

Problem 7. Eat The Cake!. Now the problem $v(x)$ such that

$v(x_0) = sup_{\{ct+1\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t log(c_t)$

S.T $c_t \in [0, x_t]$

$x_{t+1} = R(x_t - c_t)$

x_0 given

- Write the Bellman and functional equation of the problem
- Proof that the next equation is the solution to the Bellman equation

- $f(x) = \frac{\beta}{(1-\beta)^2} log(R\beta) + \frac{1}{1-\beta} log(1-\beta) + \frac{1}{1-\beta} log(x)$

- Starting with $v_0 = 0$ iterate numerically to find the Bellman equation solution.

Problem 8. Euler equation, Consider the next problem of consumption

$v(x_0) = sup_{\{c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$

s.t.

$c_t \in [0, \bar{c}_t(x_t)]$ (liquidity restriction)

$x_{t+1} = \widetilde{R}_{t+1}(x_t - c_t) + \widetilde{y}_{t+1}$

x_0 given

- Write the Bellman equation
- Derive the Euler equation

Problem 9. Given the simplified version of the model (RBC Romer) with uncertainty and infinite horizon, assume also that:

There is no growth in the population: $n = \bar{N} = 0$

Technology does not have a trend: $g = \overline{A} = 0$

Let $V(K_t, A_t)$, the value function, be the expected present value from the current period forward of lifetime utility of the representative individual as a function of the capital stock and technology.

- Explain intuitively why $V(.)$ must satisfy

- $V(K_t, A_t) = Max_{C_t, l_t}\{[lnC_t + bln(1 - l_t)] + e^{-\rho}E_t[V(K_{t+1}, A_{t+1})]\}$

Given the log-linear structure of the model, let us guess that $V(.)$ takes the form of

$V(K_t, A_t) = \beta_0 + \beta_k lnK_t + \beta_A lnA_t$, where the values of the β's are to be determined. Substituting the facts leads to $K_{t+1} = Y_t - C_t$ and $E_t[lnA_{t+1}] = \rho_a$, combining with the Bellman equation:

$V(K_t, A_t) = Max_{C_t, l_t}\{[lnC_t + bln(1 - l_t)] + e^{-\rho}[\beta_0 + \beta_k ln(Y_t - C_T) + \beta_A \rho_A lnA_t]\}$

- Find the first order conditions for C_t. Show that it implies that $\frac{C_t}{Y_t}$ does not depend on K_t or A_t

- Fin the first order condition for l_t. Use this condition and the result in the former bullet to show that l_t does not depend on K_t or A_t

- Substitute the production function and the results in the former bullets for the optimal C_t and l_t into the equation above for $V(.)$, and show that the resulting expression has the form

$V(K_t, A_t) = \beta_0' + \beta_k' lnK_t + \beta_A' lnA_t$

- what must β_k and β_A be so that $\beta_k' = \beta_k$ and $\beta_A = \beta_A'$?

- What are the implied values of $\frac{C}{Y}$ and l? are the same that we found in class for the case $n = 0 = g$?

Problem 10. Consider the same Simplified Model with uncertainty and infinite time, however , that the instantaneous utility function, u_t, is given by $u_t = lnc_t + b(1 - l_t)^{1-\gamma}/(1 - \gamma)$ as in problem 2

- Find the firs order conditions that relates current leisure and consumption, given the wage
- With this change in the model, is the saving rate s still constant?
- is leisure per person $(1 - l)$ still constant?