

QUIZ 2, PUBLIC GOODS REVISITED

Part 1. Consider the same problem we saw on lecture, Sam and Ryan are roommates. They have one day to spend between cleaning and leisure. Cleaning produces a public good, $C = C_S + C_R$, Assume the utilities are given by $U_i = \ln(L_i) + \ln(C)$. What is the budget constraint in this problem for each guy?

$$C_R + L_S = 1 \text{ and } C_R + L_R = 1$$

Part 2. Substitute $C = C_S + C_R$, in the utility function, but do not substitute the budget constraint. Use instead the method that we covered in discussion (Marginal rate of Substitution equals price relationship) to find the best responses, and then the Nash equilibrium.

$U_s = \ln(L_s) + \ln(C_s + C_R)$, then $MRS_S = \frac{L_S}{C_s + C_R} = 1$, thus $L_s = C_s + C_R$, using the budget line, $2C_S + C_R = 1$, thus $C_S = \frac{1 - C_R}{2}$, By symmetry $C_R = \frac{1 - C_S}{2}$. Thus the Nash Eq $C_S = 1/3$ and $C_R = 1/3$

Part 3. Find the efficient quantity of cleaning, (hint: we did this several times las discussion)

Here we need to add the marginal rate of substitutions $SMRS = \frac{L_s + L_R}{C}$, and $SMRS = 1$, thus $L_S = C - L_R$, Hence adding the budget lines, $L_S + L_R + C = 2$, thus $C = 1$

Part 4. $\sum_{t=0}^{\infty} \frac{A}{\beta^t} = ?$

Remember that $S = A + \frac{A}{\beta} + \frac{A}{\beta^2} \dots$ thus $\frac{S}{\beta} = \frac{A}{\beta} + \frac{A}{\beta^2} \dots$. Therefore $S - \frac{S}{\beta} = A$, or $S = \frac{A}{1 - \frac{1}{\beta}}$